How Long Is that Polygon?: A Centerline Algorithm E. I. Schaefer[1] and A. S. McEwen[1], [1]Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721 USA (schaefer@lpl.arizona.edu).

Introduction: In planetary science, we frequently encounter geomorphic features that are polygons in planform: channels, valleys, ripples, yardangs, etc. We often quantify these features with "lengths" and "widths", yet neither of these measurements is straightforward for any but the simplest polygons.

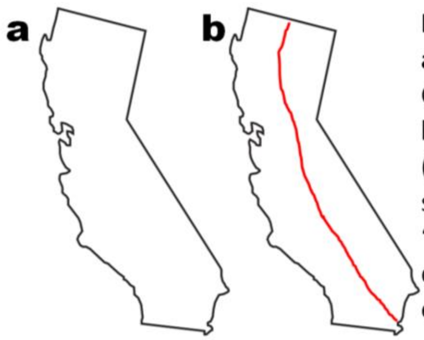

Fig. 1: **a**) Outline of California. **b**) Same as (a), with schematic "centerline" drawn by eye.

As an illustrative example, consider the outline of the state of California [**Fig. 1a**]. The human eye can easily recognize that this shape is elongate in approximately the NNW direction, but several questions immediately arise:

- Which is a better measurement of length: the simpler and shorter eastern border? the effectively fractal western coastline? neither?
- How can one measure the width of California, whether overall or at any point along its length?
- How can any measurement of length or width (or sinuosity, etc.) be reproducible if these measurements are fundamentally in the eye of the beholder?

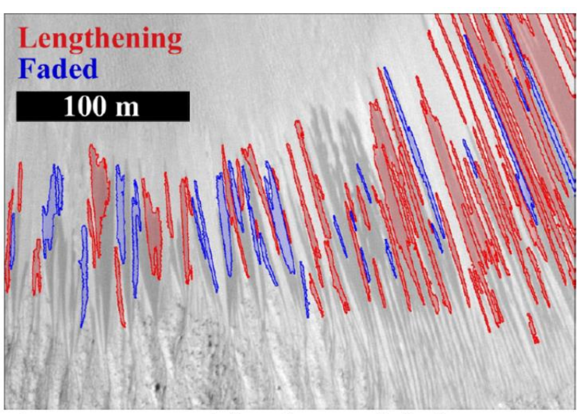

Fig. 2: automated classification [2] of RSL growth and fading regions relative to earlier HiRISE [3] image (not shown)

In the current era of abundant remotely sensed data and automated classification, yet another question arises:

- How can these and related measurements be made efficiently, preferably with a quantifiable error?

For example, the outlines of thousands of rivers on Earth [1] or recurring slope lineae (RSL) on Mars [2; **Fig. 2**] can be automatically mapped across images spanning years of changes, but this effectively leaves the scientist with only a measurement for area when a host of other measurements [4] would be useful:

- length and width
- the topology of these networks
- sinuosity
- changes over time and space for each of the above quantities
- the longitudinal topographic profile

Fortunately, the answer to each of the aforementioned questions is the same: an objective, automatically derived "centerline"—a curvilinear axis that is everywhere parallel to the length of the polygon [**Fig. 1b**]. Here, we describe our implementation of an algorithm to derive such a centerline for any polygon.

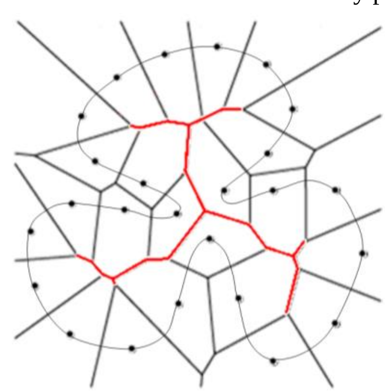

Fig. 3: Each Voronoi cell (black lines) is that region closer to one sample point (black dot) than to any other. The medial axis is colored red. [6]

Methods: [5] describe a very fast algorithm for converting a polygon to a linear representation. Called the medial axis transform (MAT), it involves dense sampling of a polygon's boundaries followed by Voronoi (Thiessen) analysis of these points and spatial filtering to isolate those facets of Voronoi cells that are wholly enclosed by the polygon [**Fig. 3**]. Unlike its predecessors, MAT's Voronoi analysis is point-based rather than line-segment-based, making it very efficient, yet it can be rigorously shown to converge on the true mathematical "skeleton" (a topological concept closely related to the centerline) [5].

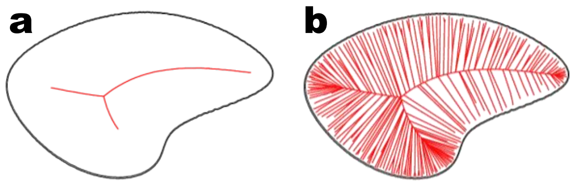

Fig. 4: Medial axes of (a) a smooth polygon and (b) the same polygon with short wavelength noise. [6]

Nonetheless, MAT has two key weaknesses.

- MAT is very unstable to noise [6]. For example, short wavelength undulation in a polygon boundary results in a very "hairy" skeleton [**Fig. 4**].
- MAT cannot reproduce the skeleton/centerline at the ends of an elongate polygon [3].

The latter issue is especially problematic if only a short portion of a elongate polygon, such as a valley, is in the field of view, or if length changes are a major focus of the study, such as for RSL [2,4].

We overcome the "hairy" skeleton problem by adapting the pruning method described by [6] and extending their pruning criteria. Similar pruning is also used to remove the edge effects of the MAT. We then use a novel bisection algorithm to reconstruct the centerline in these terminal regions.

When complete, our algorithm will have broad functionality, including:

- able to handle polygons with and without holes
- support for tuning how rounded the turns in the centerline are
- multiple options for isolating the "backbone" of a skeleton (for example, the main trunk in a map of tributaries)

Results: The algorithm is still in development, but nearing completion. Example output from the current code demonstrates the generality of the algorithm and the success of its novel pruning and centerline reconstruction components [**Fig. 5**]. The algorithm is also highly optimized:

- uses the Qhull library for Voronoi analysis
- leverages spatial indexing rather than geometric calculations wherever possible
- uses a custom geometric library specifically designed for performance

References: [1] Bryk A. and Dietrich W. (2014) *AGU Fall Meeting 2014*, EP51C-3544. [2] Stillman D. E. et al. (2015) *LPSC 46*, Abstract #2669. [3] McEwen A. S. et al. (2007) *JGR 112*, E05S02. [4] Schaefer E. I. (2015) *LPSC 46*, Abstract # 2930. [5] Brandt J. W. and Algazi V. R. (1992) *CVGIP: Image Understanding*, *55, No. 3*, 329-338. [6] Attali D. and Montanvert A. (1996) *Intl. Conf. on Image Processing 1996*, *3*, 13-16.

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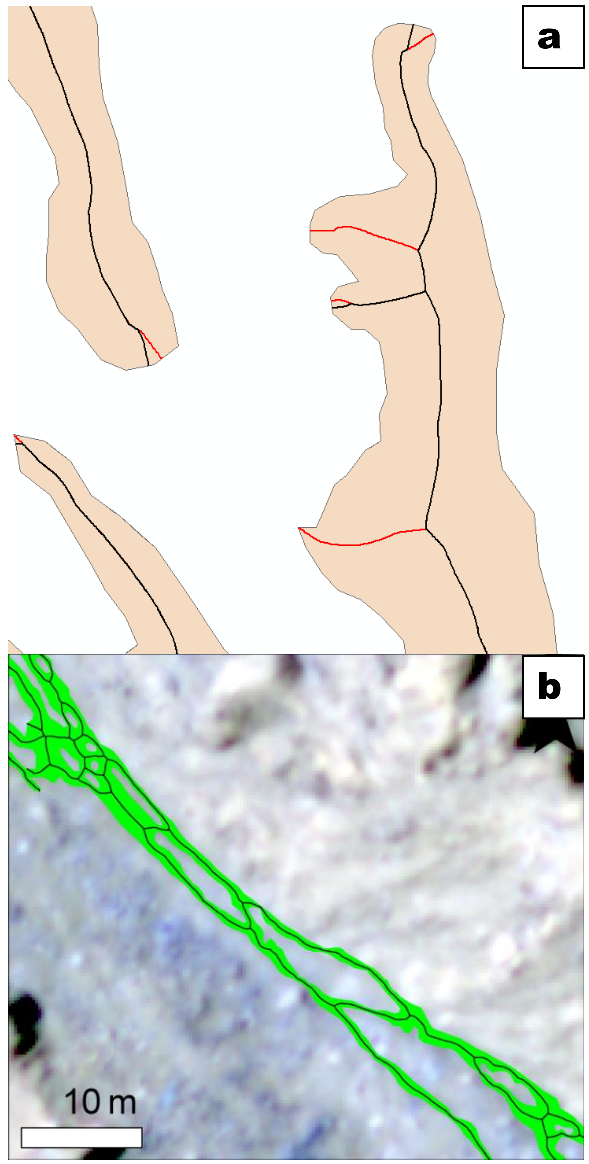

Fig. 5: (a) Centerlines (one black, the other red and beneath) for a test polygon, demonstrating how user-specified parameters affect the results. (b) A complex RSL polygon, as mapped by [4], and its calculated centerline.