

Spacecraft Attitude Determination from Landmarks on Space Objects

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ABSTRACT

Three axis attitude determination of a space vehicle is performed out using directional vector measurements. A methodology using line-of-sight vectors to landmarks, as sensed by an on-board camera, as the directional references for attitude determination is proposed in this paper. In case the distinctive features of planets or large space objects are sufficiently well known, a reference template of landmarks can be constructed *a priori*. The landmark pattern in the image captured by the camera will be warped due to perspective projection, depending on the viewing geometry and relative position of the camera. Camera pose (position and orientation) is recoverable from images if sufficient number of landmarks are observed. Here, we assume the knowledge of orbital position (translation) of the camera to determine its attitude (orientation/rotation). The positional knowledge is further utilized to reduce the search space within the catalog of landmarks. We make use of the fact that the angular separation between directed vectors connecting a pair of landmarks to the camera does not depend on camera attitude, but only on its position. Once the landmarks are identified by angular separation match, the landmark directions from camera center of perspective in reference frame as well as in the camera frame, are readily found. If more than one such vector directions are available, the attitude corresponding to the rotational transformation of reference frame to camera fixed frame can be extracted. The absolute three axis attitude updates so obtained can either supplement other sensors or serve as a back-up option in case of failure of conventional attitude sensors on-board a spacecraft due to degradation at its end-of-life.

1. INTRODUCTION

Attitude Determination (AD) is an essential pre-requisite for attitude control, which in turn decides the success of most of the Space Debris Mitigation applicable for orbital stages or spacecraft. This includes on-orbit collision avoidance, post mission disposal by de-orbiting/re-orbiting out of crowded orbital regimes, controlled or semi-controlled re-entry into atmosphere. Attitude knowledge is also important for activities such as space based surveillance of Resident Space Objects with imaging sensors, Active Debris Removal by docking space-tug with target object, etc. Three axis attitude of a satellite is usually determined from a set of vectors observed in sensor-fixed frame with the help of suitable sensors, capable of directional vector measurements, the directional references being known *a priori*. Distinctive features or landmarks on Earth, Moon or other planets, radio or optical beacons can serve as known references for attitude determination. Examples of such sensors include Magnetometers, Star sensors which provide measurement of Earth's magnetic field and stellar direction vectors, respectively, in body/sensor fixed frame. Since the directions of the vectors relative to a reference frame are known based on well established models, the attitude of the platform carrying the sensor with respect to this reference frame can be determined from the corresponding pairs of reference and sensed directions [1]. The Earth and its neighboring planets have been already extensively mapped by various remote sensing and cartographic missions. The distinctive features on planetary surfaces are sufficiently well known to establish a reference template of landmarks. In case of space objects, the corners or fiducial marks can serve as landmarks. Now, the landmark pattern in the sensed image as captured by the camera gets warped due to perspective projection for a given viewing geometry. The change in relative distance, as may arise due to altitude or platform height variation, results in changes in scale. Hence, attitude and orbital position are major contributing factors behind geometric distortion of the image sensed by an on-board camera. Conversely both can be recovered from the sensed images. At least two vectors are necessary to find three independent parameters representing the aforementioned attitude.

Although there are large number of publications on image based motion estimation for autonomous navigation, most of them are in the context of photogrammetry [2], [3], the problem of estimating camera/sensor pose (position and orientation) from corresponding ground point and their images is known within the photogrammetry community as resection problem. Other applications concern robotic exploration, rendezvous and docking, planetary descent and landing [4],[5],[6],[7], and often deal with determination of orbit rather than attitude. There are relatively few publications where the problem of attitude determination of a satellite through feature detection is specifically

addressed. Abidi and Chandra [8]. propose a pose estimation technique based on quadrangular targets. Their algorithm extracts camera interior parameters and pose using a geometric technique. Mukundan and Malik [9] use monocular images and their normalized central moments (independent of scale and orientation) stored in a look-up-table (LUT) for all possible viewing angles. Perrier [10] uses an image registration method with several pushbroom cameras where the image is treated as concatenation of a series of 1-D images. Bamber et al. [11] used a pair of canted push broom sensors (namely, port and starboard cameras) with region of overlap. Effects of attitude pointing and rates manifest as shifts between row and columns of image which is found in the Fourier domain by correlation technique to build a model, and finally attitude is extracted by model inversion of observed shifts.

In Mukherjee et al. [12], the knowledge of orbital position (translation) is assumed to be available, and the camera attitude (rotation) alone is estimated. The positional knowledge is further utilized to restrict the search space within the reference catalog of landmarks to a reduced, putative set for establishing correspondence between feature points in the reference and the sensed images. Possible limitations of this image based approach are the complexities that may arise in handling partial features due to cloud cover, or lack of features over oceanic surfaces. But this limitation will be absent in case of atmosphere-less planetary bodies like moon/ asteroid. Landmark identification under different illumination condition and viewing angles are still challenging, but can be expected to be overcome with advent of modern image processing techniques.

The paper is organized as follows. Section 2 begins with the assumptions, preliminary mathematical formulations and the solution approach, which is further elaborated in the ensuing sections. Section 4 describes the landmark viewing geometry, followed by landmark identification methodology in Section 5. Attitude determination from LOS directions towards landmarks are described in section 6. Finally simulations results are presented in Section 6 and the paper concludes in Section 7.

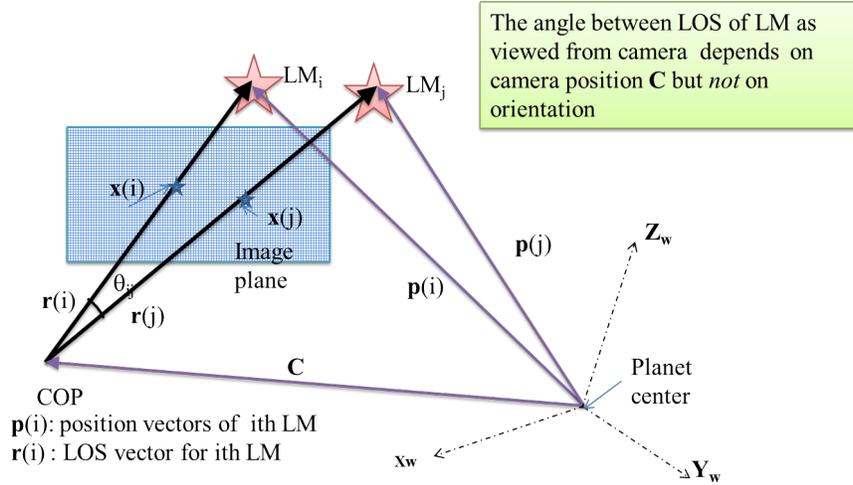
2. PROBLEM FORMULATION AND SOLUTION APPROACH

The problem of landmark based attitude determination is formulated as: Find the 3×3 proper orthogonal attitude matrix \mathbf{R}_{IC} which maps the line of sight vectors connecting landmarks to COP (center of perspective) in reference frame \mathcal{F}_I to those in camera frame \mathcal{F}_C . The following assumptions are made:

- The landmarks are points (namely corners, intersections etc.)
- The position of the landmarks in a reference frame are known.
- The camera is a frame camera where pin-hole camera model under central perspective geometry is applicable.
- Position information of the satellite, $\tilde{\mathbf{C}} = [X, Y, Z]^T$, at the imaging epoch t is known.
- The camera is calibrated, hence intrinsic parameters, namely focal length (f), pixel size (s), and principal point offsets (u_0, v_0) are known. Further, let: $n_p = 1/s$; $\tilde{f} = f/s = fn_p$, hence the camera matrix:

$$\mathbf{K} = \begin{bmatrix} \tilde{f} & 0 & u_0 \\ 0 & \tilde{f} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

\mathbf{R}_{IC} , the camera attitude, can be found from the directed vectors connecting landmarks to the camera, provided their directions are also known in the reference frame. This calls for a catalog of landmarks containing their positional information to be available *a priori*, so that the reference directions to landmarks from the camera center can be computed as the position of the camera is assumed to be known. As discussed in details in Section 3, the angle between the vectors joining camera center to landmark pair do not depend on camera attitude, same pair of landmarks will have the same directed angle between them provided camera position is unchanged. Then a technique similar to that adopted for finding angle-only match in star sensor [13] is utilized to match the observed landmarks with cataloged landmarks. Obviously, a catalog of all possible landmarks, as such, will be huge and establishing correspondence with all such inter-landmark combinations will be overly expensive. However, it is obvious that for a given camera position, the entire catalog need not be searched, only those landmarks within the admissible zone can be picked up as candidate landmarks for matching. If more than one pair of such vector directions in reference frame and observed camera fixed frame are available, attitude matrix that transforms camera fixed frame to reference frame can be found.



If $C, p(i), p(j)$ are known, θ_{ij} can be found as angle between LOS vectors $r(i), r(j)$ in world frame

Fig. 1. Viewing geometry of LM pair

3. LANDMARK VIEWING GEOMETRY

Let $x_i = [u_i, v_i, 1]^T$ be the homogeneous coordinates [14] of the image of a landmark LM_i as captured by a camera with intrinsic matrix K . Then, the viewing direction vector from camera to landmark LM_i in camera fixed frame is given by,

$$\tilde{r}_i = K^{-1}x_i \tag{2}$$

Let θ_{ij} be the 3D viewing angle subtended at the camera center by the i th and j th landmarks (LM_i and LM_j), this is the angle between corresponding view direction vectors connecting the camera center to the respective landmarks (see Fig. 1).

$$\cos \theta_{ij} = \frac{x_i^T K^{-T} K^{-1} x_j}{\sqrt{x_i^T K^{-T} K^{-1} x_i} \sqrt{x_j^T K^{-T} K^{-1} x_j}} \tag{3}$$

where, $K^{-T} = (K^T)^{-1} = (K^{-1})^T$ can be readily computed. Let the i th landmark vector be \tilde{p}_i and camera position be \tilde{C} in the same reference frame \mathcal{F}_I . The ray direction from camera to landmark as viewed in this reference frame, \tilde{r}_i , is also known.

$$\begin{aligned} \tilde{r}_i + \tilde{C} &= \tilde{p}_i \\ \Rightarrow \tilde{r}_i &= [\mathbf{I}_3 | -\tilde{C}] \mathbf{p}_i \end{aligned} \tag{4}$$

where, $\mathbf{p}_i = [\tilde{p}_i^T, 1]^T$ denotes the homogeneous coordinates. In camera frame $\tilde{r}_{ci} = \mathbf{A}_{IC} \tilde{r}_i$, where, \mathbf{A}_{IC} denotes camera attitude relative to \mathcal{F}_I . Let θ_{ij} be the angle between the two landmarks as viewed from camera positioned at \tilde{C} ¹ (refer Fig.1). Then,

$$\cos \theta_{ij} = \frac{\tilde{r}_i \cdot \tilde{r}_j}{\|\tilde{r}_i\| \|\tilde{r}_j\|} = \frac{\tilde{r}_{ci} \cdot \tilde{r}_{cj}}{\|\tilde{r}_{ci}\| \|\tilde{r}_{cj}\|} \tag{5}$$

$$\text{Let } \gamma_{ij} \triangleq \tilde{r}_i \cdot \tilde{r}_j = \mathbf{p}_i^T \mathbf{p}_j - \tilde{C}^T (\mathbf{p}_i + \mathbf{p}_j) - \tilde{C}^T \tilde{C} \tag{6}$$

$$\cos \theta_{ij} = \frac{\gamma_{ij}}{\sqrt{\gamma_{ii} \gamma_{jj}}} \tag{7}$$

¹here we use $\tilde{()}$ to denote representation in homogeneous coordinates [14]

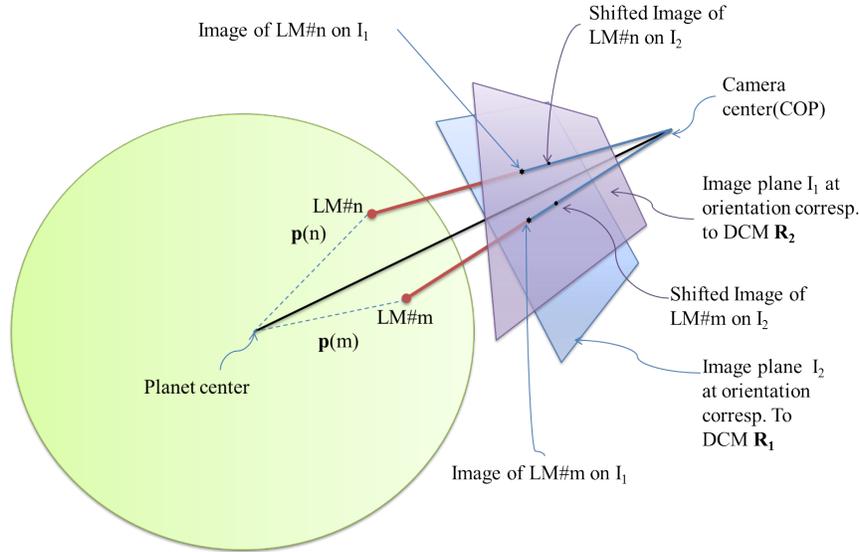


Fig. 2. Same pair of landmarks viewed from different orientations but the same camera position

Thus, the angle between the rays from camera center to the landmark pair changes with the camera position \tilde{C} , but it does not depend on its attitude, the attitude matrix \mathbf{A}_{IC} does not appear in expression for θ_{ij} in Eq. 5. This is illustrated in Fig. 2. In other words, the same pair of landmarks will have the same angle subtended by them at the COP when viewed from different attitude, as long as \tilde{C} remains unchanged.

4. LANDMARK IDENTIFICATION

A Master Catalog containing the positional coordinates of all landmarks is constructed *a priori* as a static database. Then, using the instantaneous position knowledge of camera \tilde{C} at the imaging capture epoch, a sub-catalog is built dynamically, which contains the landmarks likely to be captured by the camera at its current position. The angular separation (θ_{ij}) between all possible pairs within this substantially reduced set of landmarks are computed in terms of direction cosines and also stored in this sub-catalog. The goal of landmark identification is to uniquely for each landmark in the image its corresponding cataloged counterpart. This is illustrated in Fig. 3 for a simple case of 4 landmarks. Here, let the landmarks corresponding to cataloged index M, K, G, P are captured in image frame and (shown in top left) be indexed as 1, 2, 3 and 4. In other words the landmarks from \mathcal{F}_I to pixel frame \mathcal{F}_P are mapped as:

$$M \rightarrow 1, K \rightarrow 2, G \rightarrow 3, P \rightarrow 4$$

Given the images of landmarks, one needs to retrieve the correct indices from sub-catalog. We now describe the identification method. Let there be N landmarks captured in an image. We pick observed LM#1 as pivot. Then cosine of angular separation of the pivot with other observed landmarks, say #2, #3... are found from the inner product of corresponding LOS (Line-of-sight) vectors. Let a_{1j} be the cosine of angle between the first landmark and the j th landmark:

$$a_{1j} = \mathbf{w}_1^\top \mathbf{w}_j \tag{8}$$

assuming all the observed vectors to be normalized. Now, the central idea of the matching algorithm is to find a cataloged landmark (called the pivot) such that, the angular distances to its neighbors are most similar to a given LM and its neighboring landmarks captured in the image. For each inter-landmark angle $a_{1j}, j = 2, 3, 4 \dots$, those landmark pairs $\{p, q\}$ from sub catalog are picked whose included angles α_{pq} are within a range $a_{1j} \pm \delta$, with δ being the tolerance band. Hence, the selection criterion is

$$|\alpha_{pq} - a_{1j}| \leq \delta \tag{9}$$

Needless to say, many pairs $\{p, q\}$ in the sub-catalog will meet this matching criterion. Let this set of admissible matches be denoted by A_{1j} . Even in case there is a unique pair, it is not possible to say whether the observed LM#1

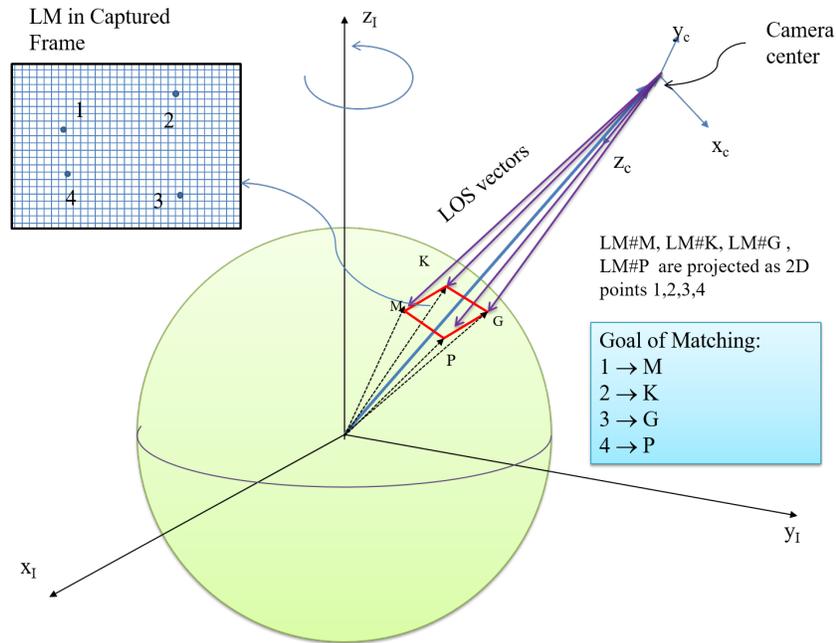


Fig. 3. Illustration of landmark identification

matches p or q within the catalog. By choosing different neighboring j we get different sets A_{1j} , $j \neq 1$. The common index in the intersection of all sets A_{1j} , namely $m = \bigcap A_{1j}$ will give the matching catalog index of LM#1, provided the set $\bigcap A_{1j}$ has just one entry (a singleton set). This is illustrated in Fig. 4.

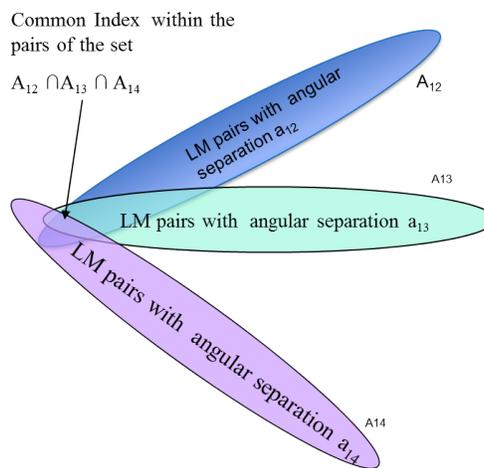


Fig. 4. Pole technique for LM identification

Once the observed LM#1 is identified with a catalog landmark, with index m , the process is repeated to identify the remaining landmarks.

5. ATTITUDE DETERMINATION FROM DIRECTIONAL VECTORS TO LANDMARKS

Let $\mathbf{x}_k = [u_k, v_k, 1]^T$ be the homogeneous vector corresponding to the k th landmark being imaged as a point with inhomogeneous line and pixel coordinates as (u_k, v_k) . This vector can be converted to the camera frame \mathcal{F}_C by

intrinsic matrix \mathbf{K} as,

$$\mathbf{w}_k = \mathbf{K}^{-1}\mathbf{x}_k; \tag{10}$$

Then reference to camera quaternion \mathbf{q}_{IC} is computed using point solution of attitude from the set of corresponding sets of vector pairs, namely $\{\mathbf{r}_k, \mathbf{w}_k\}$. A novel method, Direct ESTimator of Euler axis and ANgle (DESTEAN) [15] is used for finding the attitude in Euler principal axis and principal angle parameterization (\mathbf{e}, θ) . The method is based on interpretation of Euler Axis of rotation as orthogonal to the difference of reference and observed directions. The formulation stems from the property that the rotation axis \mathbf{e} makes the same angle α_i with the reference vector \mathbf{u}_i in \mathcal{F}_I and observed vector \mathbf{v}_i in \mathcal{F}_C , and hence is perpendicular to the difference between the original and the transformed vectors:

$$\cos \alpha_i = \mathbf{v}_i^\top \mathbf{e} = \mathbf{u}_i^\top \mathbf{e} \Rightarrow \mathbf{d}_i^\top \mathbf{e} = 0 \tag{11}$$

$$\mathbf{d}_i \triangleq (\mathbf{v}_i - \mathbf{u}_i) \tag{12}$$

For single observation, the Euler axis \mathbf{e} cannot be found unambiguously, as it can lie anywhere in the 2D plane Π_1 orthogonal to \mathbf{d}_1 , refer Fig. 5. With another pair of vectors $\mathbf{u}_2, \mathbf{v}_2$, the rotation axis can be found along the

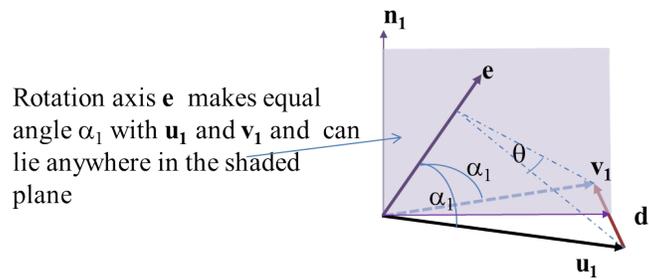


Fig. 5. Single vector observation: \mathbf{e} lies on the shaded plane normal to \mathbf{d}_1

intersection of the two planes Π_1 and Π_2 , normal to \mathbf{d}_1 and \mathbf{d}_2 respectively (unless the planes themselves become parallel leading to a degenerate condition), by solving

$$\mathbf{M}\mathbf{e} = 0 \tag{13}$$

where, $\mathbf{M} = [\mathbf{v}_1 - \mathbf{u}_1 \mid \mathbf{v}_2 - \mathbf{u}_2]^\top = [\mathbf{d}_1 \mid \mathbf{d}_2]^\top$. For multiple observations, \mathbf{e} is the line of intersection of $\Pi_1, \Pi_2 \dots \Pi_n$, found as solution to the linear homogeneous Eq. 13 with $n \times 3$ matrix \mathbf{M} given by,

$$\mathbf{M} = [\mathbf{d}_1 \mid \mathbf{d}_2 \mid \dots \mid \mathbf{d}_n]^\top \tag{14}$$

In practice, due to observation noise, the planes Π_i containing the vectors will not intersect along a unique line. The Euler principal axis of rotation, \mathbf{e} is found by solving homogeneous least-square Eq. 13, as the last column of the right singular vector after SVD of \mathbf{M} [16].

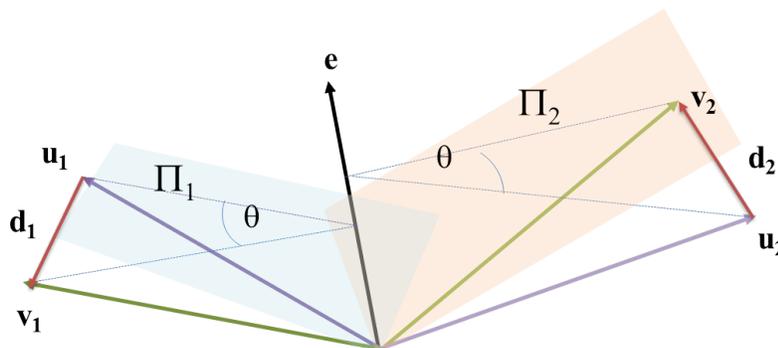


Fig. 6. Two vector observations: \mathbf{e} along the intersection of planes Π_i and Π_2 which are normal to \mathbf{d}_1 and \mathbf{d}_2 respectively

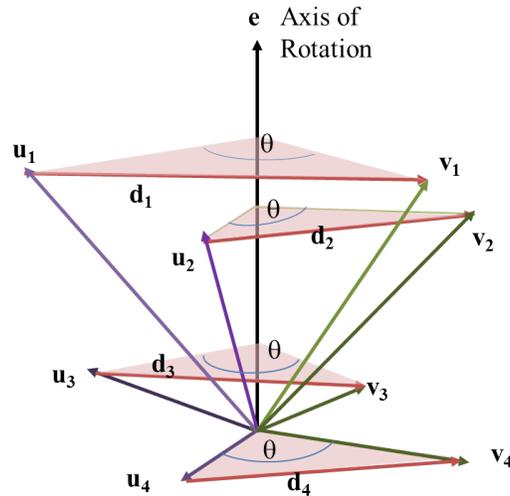


Fig. 7. Multiple vector observations: Euler principal axis of rotation, e is normal to all difference vectors, d_i 's

Next, using Rodrigues' rotation formula to express DCM R in terms of Euler axis e and angle θ , and rearranging terms from $v_i = Ru_i$, one can obtain

$$(-e^\times u_i) \sin \theta + (I_{3 \times 3} - ee^\top)u_i \cos \theta = v_i - ee^\top u_i \tag{15}$$

All n observations are concatenated² to form a set of linear equation of the form $Ax = b$, with $x = [\sin \theta, \cos \theta]^\top$. Hence, θ is recovered using 2-argument arc-tangent after solving for x .

When all of the d_i vectors are parallel to each other, so are the planes Π_i , and the line of intersection can no longer be found. A method similar to Method of Sequential ROTation (MSROT) due to Shuster [1] is invoked to handle the degeneracy.

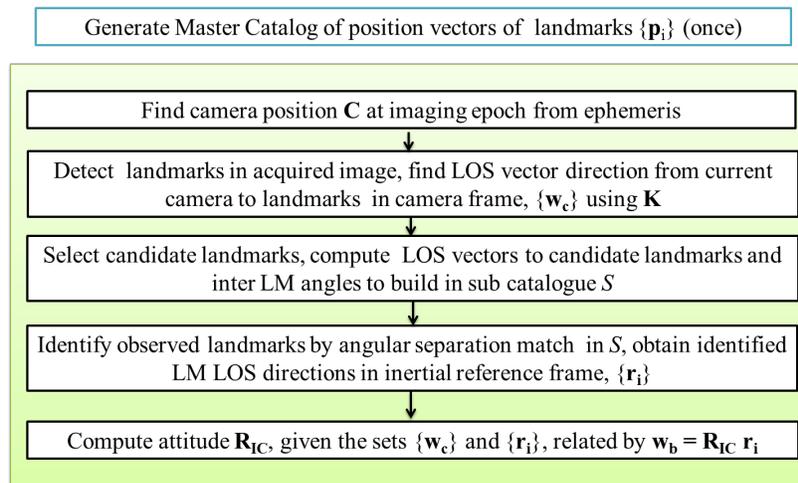


Fig. 8. Outline of the proposed landmark based algorithm

² $()^\times$ is used to denote the 3×3 skew symmetric matrix associated with the cross product of 2 vectors a and b , which can be used to pre-multiply the vector b to compute the cross product by matrix multiplication: $a \times b = a^\times b$

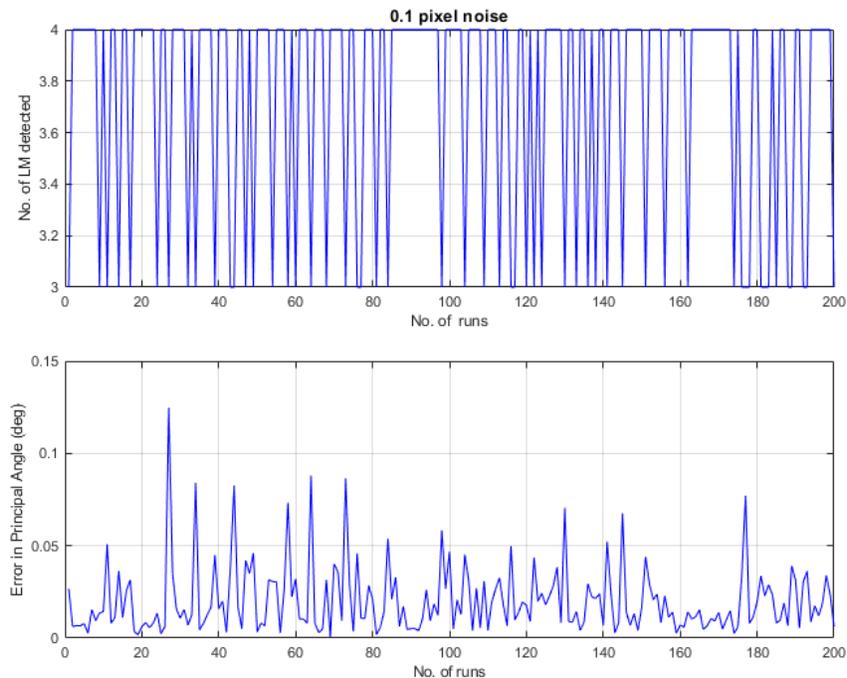


Fig. 9. Estimation error in principal angles

6. SIMULATION RESULTS

A simulation was carried out to validate the proposed method. A satellite at 100 km polar, circular Lunar orbit is considered as the platform. With T_{ims} as the image start epoch, the Cartesian position vector $\tilde{\mathbf{C}}$ at epoch $T_c = T_{ims}$ is extracted from its ephemeris. Using an ideal nadir pointing attitude DCM (Direction Cosine Matrix) \mathbf{A}_{IB} and known mounting matrix \mathbf{A}_{BP} , the ideal attitude of the camera in J2000 frame is constructed. A master catalog of landmarks is prepared beforehand. The expected landmarks captured within the FOV of an on-board camera are simulated by back projecting landmarks on the camera focal plane, then the 2D image coordinates are corrupted by adding random noise. With this synthetic landmark images, the algorithm, as outlined in Fig. 8, is tested to identify the landmarks and subsequently extract the attitude as $\hat{\mathbf{q}}_{IC}$. The process is repeated after adding a time offset ΔT to image start to get time of frame capture $T_c = T_{ims} + \Delta T$, and also for different sets of \mathbf{A}_{IB} , corresponding to different yaw, roll, pitch errors. \mathbf{q}_{IC} being the true camera attitude, the error quaternion is computed as

$$\Delta \mathbf{q} = \mathbf{q}_{IC}^* \otimes \hat{\mathbf{q}}_{IC} \quad (16)$$

The error between estimated attitude $\hat{\mathbf{q}}_{ic}$ and ideal attitude used to simulate the landmarks, \mathbf{q}_{IC} , is found in terms of Euler principal angle. The corresponding estimation error in terms of Euler principal angle is given by $\theta_e = 2 \arccos(\Delta q_4)$. (See Fig. 9), where the line and pixels were corrupted by 0.1 pixel error. The estimation error is found to be within 0.1 deg in most of the cases.

7. CONCLUSIONS

The results obtained with synthetic images generated with frame camera model have been promising. The methodology needs to be tested on real image acquired by frame camera. It may be noted that unlike the cases where rotation is extracted from Homography [14], the landmarks need not be co-planar. Although the image processing and landmark identification are computationally expensive, given the present trend of advances in space technology and ongoing demonstrations of spacecraft autonomy, more and more complex processing is becoming realizable on board. Hence, the proposed on-board image based method can be especially useful where the camera based instruments remain functional even after a decade of on-orbit operation but some of the attitude sensors lose their functionalities due to aging. If imaging payload itself can be used as an attitude sensor, considerable benefits can be reaped in terms of size and mass which are highly desirable for low cost, small satellites.

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