

BAS3E: A framework to Conceive, Design, and Validate Present and Future SST Architectures

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ABSTRACT

One of the main missions of a Space Surveillance system is the detection and cataloguing of resident space objects (RSO) having a size compatible with the detection constraints of its sensors. While radars are used to observe objects placed at Low Earth Orbits (LEO), and telescopes to observe objects orbiting in Medium (MEO) and Geostationary Orbits (GEO), for objects orbiting in Highly Elliptical Orbits (HEO), both types of sensors are suited for observations.

In addition of the pure observation of the RSO by a given sensor, the correlation of the acquired observation to an existing or non-existing object on the catalogue, the initial orbit determination and the improvement of the orbit accuracy to maintain a catalogue of sufficient accuracy are activities of key importance in order to be in a position to provided added-value services from the catalogue.

Additionally, and in order to optimize the observational resources for catalogue build-up and maintenance as well as for the provision of the added value services, the tasking and scheduling of a distributed network of sensors that may or may not be coordinated is also of paramount importance.

The goal of this paper is to introduce a complete SST simulation Framework developed by CNES with the goal to evolve existing Space Surveillance and Tracking (SST) network, both from a software and hardware point of view, and to define major evolutions of existing SST networks.

The simulation framework presented in the paper is called BAS3E, and works both with simulated or real observational data. It implements the capability to simulate ground and space based sensors via the integration of the following functions:

- Detection, tracking and generation of observations of space objects,
- Object identification and tracking correlation,
- Orbit determination,
- Maintenance of a space debris catalogue,
- Centralized / de-centralized tasking and scheduling.

In addition to the introduction of the simulation framework, results of the main constitutive functions will also be presented on a test case illustrating the design of a tracking radar network.

1 BAS3E main features

1.1 Tool architecture

BAS3E (**B**anc d'**A**nalyse et de **S**imulation d'un **S**ysteme de **S**urveillance de l'**E**space – Simulation and Analysis Bench for Space Surveillance System) is a CNES-owned tool developed in JAVA and based on the following CNES libraries:

- PATRIUS: core space dynamics Java library featuring all the main domains of space dynamics (Dates, frames, orbit propagation, attitude, etc.). PATRIUS is extensively validated, used in operational flight dynamics subsystem, and provided as open source (<https://logiciels.cnes.fr/en/content/patrius>)
- BIBOR: measurements (both simulated or real observational data) and orbit determination Java library. BIBOR is used in the operational software suite using the TAROT telescope network [1] to build the French autonomous GEO catalogue

From the beginning, BAS3E has been designed and developed for parallel computing in, for instance, a High Performance Computing (HPC) service. BAS3E is generally used on CNES own HPC service, which is dedicated to scientific projects demanding a great computing and processing capacity.

1.2 Flight dynamics features

A generic BAS3E simulation is represented in Fig. 1:

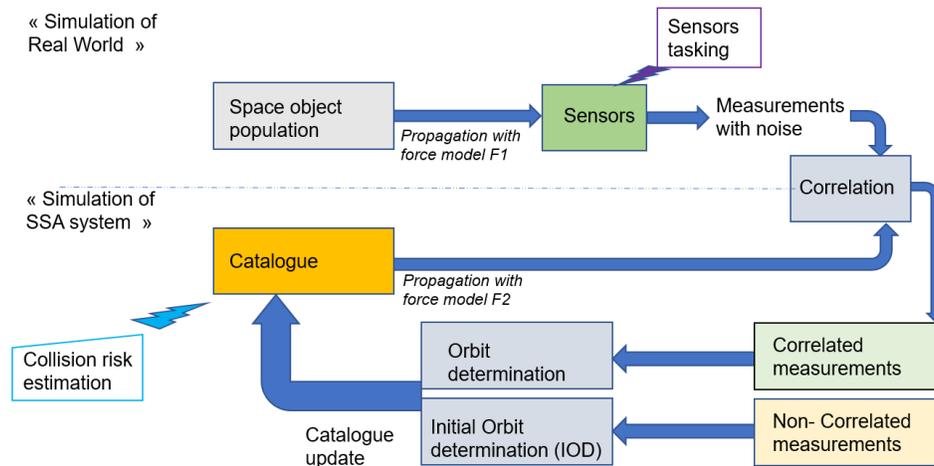


Fig. 1. Cataloged objects in low Earth orbit.

One of the key BAS3E features is its ability to separate the “simulation of Real World” and the “Simulation of SSA system”. A given space object population is propagated using a given force model F1. Those objects are observed by a network of sensors generating noisy measurements: those are the only information available for the SSA system to perform the cataloguing, which is the case in the “real world”. The catalogue of object, that may differ from the space object population, is propagated using a different force model F2 to account for the uncertainties and inaccuracies of the propagation and environment models (for example atmospheric models and the poorly known solar activity). Correlation algorithms are used to correlate the noisy measurements with objects of the catalogue: correlated measurements are used in an orbit determination process to refresh the catalogued objects orbit and covariance information, whereas non-correlated measurements are used in an Initial Orbit Determination (IOD) process to add new objects within the catalogue. Collision risks can then be estimated for all the objects of the catalogue.

The following table shows the current and possible future capabilities of BAS3E:

Theme	Main current capabilities	Envisaged future capabilities
Space object population	User-defined initial population (TLE, MASTER, ...) Object fragmentation using NASA breakup model [2].	Mega-constellation specification Non-spherical objects
Orbit propagation	Numerical or analytical propagators. Complete force model.	Maneuvers (LEOP, station keeping, collision avoidance, ...)
Sensors modelling	Ground based and space based sensors Radars (Range or Doppler) on mono or bi-static configuration. Detection based on radar cross section using NASA SEM model [3]. Telescopes (survey and tracking) detection based on magnitude computation. TDOA (Time Difference Of Arrival) measurement function	Laser ranging Probability of detection and false alarm Signal to Noise Ratio computation

Sensors scheduling	User-defined surveillance strategies. Scheduler presented in §2.2	Further improvement of the scheduler
Tracklet-to-orbit correlation	Several distances (Euclidian, Mahalanobis, ...) applied to a Nearest Neighbor scheme.	Global Nearest Neighbor algorithm. Correlation for maneuvering objects (optimal control problem).
Orbit Determination	Least Squares. Kalman Filter (extended and unscented).	Gaussian Mixture Kalman Filter (extended and unscented).
Initial Orbit Determination	Classical three pair of angles IOD methods: Gauss, Laplace[6], Gooding [7]. Tracklets linkage methods: Keplerian Integrals [8], Siminski [9] Doppler IOD [10]. Admissible regions [11].	Coupled optical-radar IOD methods for high-LEO and GTO objects.
Catalog handling	Short-term encounter collision risk [12]	Long-term encounter collision risk

Tab. 1. BAS3E capabilities

Each of these features has been validated extensively, taking advantage of the PATRIUS and BIBOR libraries already used in operational systems, while the global simulation process has been validated by modelling the French-owned and operated GRAVES (Grand Réseau Adapté à la VEille Spatiale) radar and TAROT telescopes and comparing their simulated performances (space object population coverage, catalog size, ...) with real operational data.

2 Sensor scheduling

2.1 Generalities

Resident space object (RSO) observations are performed through sensors with a field of view (FOV) that is either fixed, typically the case of survey radars, or steerable: for the latter a tasking strategy is then required to determine at any time the direction of pointing of the FOV. Due to the relatively (with respect to the needs) low observational capacities available, the optimization of such tasking is required to maximize the performance of a space surveillance system: one might want to increase the number of objects catalogued by the system or to improve the accuracy of the orbits of the catalogued objects. In particular, when several sensors are considered, coordinated tasking is preferred to reach a higher level of performance. While survey sensors are typically pointed towards specific regions of the sky to maximize the number of RSO observations, from both known and unknown RSO, tracking sensors are tasked to observe a specific RSO in order to improve the knowledge of its current and future positions. The scheduler implemented within BAS3E software solves the coordinated tasking problem of a network of tracking sensors, both radars and telescopes, taking into account the information already brought by survey sensors.

2.2 Implementation within BAS3E

2.2.1 Information gain approach

The purpose of the scheduler is to generate an observation plan for a network of tracking sensors in order to meet a predefined requirement in accuracy for the catalogued objects. An acceptable uncertainty in position is defined for each object as a target ellipsoid, or, equivalently, a target covariance in position P_{target} (square matrix of dimension 3). The observation plan built by the scheduler is compliant with the requirement if the ellipsoid associated to the expected covariance, P_{plan} , lies within the target one. This constraint reads:

$$X_i^T P_{target} X_i \leq 1, \quad i = 1:3 \quad (1)$$

where X_i is a principal axe of the expected ellipsoid.

The a priori information of an object jointly with the expected information provided by the surveillance network to that object is gathered in a generalized a priori information. This information is given by the normal matrix [13], which is the inverse of the covariance:

$$(P^{apr})^{-1} = (P_0^{apr})^{-1} + (H^T W H)_{survey}, \quad (2)$$

where H is a matrix of dimension $m \times p$ containing the partials of the observations, m is the number of measurements, p the size of the state vector and W the weight matrix of dimension $m \times m$. In general, m can be very large, but not p . The matrix $H^T W H$ is of dimension $p \times p$, a reasonable size that allows us to apply standard inversion procedures (QR decomposition, for example). The normal matrix verifies the additive property, and that property is used in the computation in the following way: the product of the $m \times p$ matrices is not applied, but the contributions of each measurement independently is summed up:

$$H^T W H = \sum_{obs=1}^m (H^T W H)_{obs}, \quad (3)$$

This procedure of computing the normal matrix is essential afterwards in the scheduling process since the contributions of several potential observation intervals can be added and subtracted to assess the effectiveness of one schedule over another. Each generated plan will contain an information:

$$P_{plan} = \left((P^{apr})^{-1} + \sum_{obs=1}^t (H^T W H)_{obs} \right)^{-1} \quad (4)$$

where t is the number of tracking observations that have been planned. This information, $\Lambda = P_{plan}^{-1}$, is also known as the Fisher information matrix. The scheduling is divided into two computation stages. The first one, called *InformationGainStage*, makes a pre-filter for those objects for which surveillance measurements are sufficient to reach the target accuracy and computes the normal equations for those objects that need additional information provided by tracking sensors. The second one is the scheduler itself. The following information is provided to the scheduler:

- A temporal horizon h ;
- For each catalogued object o :
 - its generalized a priori information, P_o^{apr} ;
 - its target covariance in position P_o^{target} . This parameter depends on the object as we could, for instance, define a restrictive accuracy for objects involved in a collision risk and a less restrictive one for the other objects in the catalogue;
 - its priority level $prio_o$. This value can be used in the scheduler to prioritize the observation of some set of objects.
- For each tracking sensor s , its characteristics (measurements frequency, measurement cost, etc.);
- For each object o and for each tracking sensor s , the visibilities that s has on o during the horizon h , represented by visibility intervals $Vis_{o,s}$

- For each visibility interval v , the normal equations associated to one or more points of the v . This can be parametrized to define as many points as possible measurements can be taken in the interval (taking into account the measurement frequency of the sensor)

2.2.2 Scheduler tool

The scheduler is an enhanced version of the one presented in [4]. The constraint-based scheduling problem to be solved involves constraints of many kinds:

- Temporal constraints: measurements can only be taken within the visibility intervals, and duration delays must be respected between successive observations
- Resource constraints: a limited number of tracking sensors is available, with a different cost of using them that can be a variable to be optimized.
- Information constraints: each observation provides a different information gain, and each object has a target covariance to be met.

Regarding the optimization criteria, the information constraints is used as the main criterion: the number of objects that satisfy the information constraint is optimized. Several variants of this criterion, by taking into account or not the priority level of the object, and also by minimizing the distance from objects to their target accuracy, have been tested. In order to tie break between solutions that are equivalent regarding the satisfaction of information constraints, the overall cost of the schedule, i.e. the sum of costs of each observation in the schedule, can be minimized.

As relative quick answers are needed in a real case sensor scheduling optimization, the scheduler is based on a parallel greedy search algorithm. A dynamic search is performed, meaning that the sorted list of candidate observations is recomputed each time an observation is inserted in the plan in order to take into account the associated information gain. Thus, observations providing a different and complementary information than the inserted ones are favored. In that way, diversity in geometric conditions is automatically encouraged and, as a consequence, the number of observations needed to reach the accuracy threshold is decreased, optimizing the overall behavior of the sensor network.

The greedy search algorithm is guided by heuristics. These heuristics can be applied on:

- The priority level of the objects: favouring the observation to those objects of higher priority,
- The distance to the accuracy threshold: a distance is defined for each object as the ratio between the volume of the expected covariance with respect to the target covariance. If that distance is lower than 1, the accuracy constraint is met. Two heuristics are available:
 1. Favor those objects whose distance is closer (but higher) than one: the effort is done for those objects that almost reach the accuracy constraint,
 2. Favor the objects with the higher distances: the effort is done for the less known objects, preventing the eventuality of not being able to track them in the future because of their large position uncertainty.
- Sensors: favoring those that are used the less, or favoring those whose operational cost is lower.

Not all the heuristics need to be applied. A *constant* possibility is always possible, in that case, the particular heuristic does not influence the choice of the greedy algorithm.

In order to enlarge the search domain of the greedy algorithm, there is the possibility to add *randomness* in the choice of the observation to be inserted into the plan. This means that each time a decision is taken it can be a choice given by the heuristic or a random choice. The exploration of the optimum solution is expanded and not limited strictly to the path guided by the heuristics.

The parallel programming is based on the master-slave paradigm. The scheduler is managed by the master that distributes different tasks to the slaves. In case of large problems (large number of objects and/or a large temporal horizon), the master divides the scheduling effort between different slaves and then schedules a plan using tasks selected by slaves. There is a constant interaction between master and slaves to circulate all the information, especially for the dynamic computation. Finally, in order to increase the search exploration, a portfolio strategy is used by running in parallel several concurrent teams. Each team is composed of a master and several slaves and is

configured with a different set of heuristics and a different percentage of randomness. A global master finally selects the most interesting schedule given the criteria of plan cost and number of object reaching the accuracy goal.

Because of the specific feature of LEO and GEO orbits and the sensors involved in their tracking, some different scheduling strategies are handled. For example, for radars observing LEO objects a minimum time for the track is required. Once an observation is inserted, the time of that observation can be extended up to cover the whole visibility interval if there is a substantial information gain. This way of proceed is not applicable to GEO objects whose visibility interval for a particular sensor can be the entire night. Hence, a maximum time for the track is applied and consecutive tracks are not permitted but, on the contrary, it is encouraged to spread them over the night. Besides, optical sensors are subject to meteorological conditions and, in order to compensate the information loss for a scheduled observation that cannot be taken because of a cloudy sky, the normal matrix are reduced by a factor in coherence with the cloud probability. Thus, more observations are scheduled following this conservative approach in order to be sure, with a certain probability, that the gathered observations will be sufficient to reach the accuracy threshold.

3 Illustration on a test case: design of a tracking radar network

3.1 Test case description

To illustrate BAS3E scheduler’s capacities, the following test case is considered:

- A LEO population is derived from ESA MASTER-2009 population: only the objects with a size bigger than 10 cm are selected which represent around 18000 objects. To ease the analysis, a representative subset (10%, 1794 objects) of the full population is selected and propagated over a week without manoeuvres.
- An initial catalogue is assumed, containing all the objects of the population with the following uncertainties:
 - Initial uncertainty on position consistent with TLE uncertainties [5]
 - Relative Gaussian uncertainty of 20% on area to mass ratio
- Uncertainty about the dynamical model is accounted by using two different force models for the RSO population and catalogue propagation: a reduced earth potential development (12 by 12 vs 4 by 4) and a different atmospheric model (MSIS00 and US76) are considered
- The following sensors are simulated:

Sensor	Type	Frequency	FOV	RCS @range	Azimuth accuracy	Elevation accuracy	Range accuracy	Range Rate accuracy
“SR”	Survey radar	400 MHz	[30° by 180 °]	0.015 m ² @1000km	0.3°	0.3°	-	0.2 m/s
“LPTR”	Tracking radar	5 GHZ	[1° by 1°]	0.30 m ² @1000km	0.1 mrad	0.1 mrad	3m	-
“HPTR”	Tracking radar	3 GHZ	[1° by 1°]	0.001 m ² @1000km	0.1 mrad	0.1 mrad	3m	-

Tab. 2. Sensors characteristics

It is then studied how this initial catalogue is maintained over a week considering the following sensor networks:

Network	Composition	Comment
SN0	SR located in France mainland	Reference case to estimate the “basic performance” with no tracking radar
SN1	SN0 + 2 LPTR: 1 in France mainland, 1 in Guiana	Impact of one high performance tracking radars placed in several location
SN2	SN0 + 1 HPR in France mainland	
SN3	SN0 + 1 HPR in Guiana	
SN4	SN0 + 1 HPR in Svalbard	
SN5	SN0 + 3 HPTR in the above locations	Impact of three high performance tracking radars with coordinated tasking

Tab. 3. Sensors characteristics

Figure 2 shows the considered sensors locations:

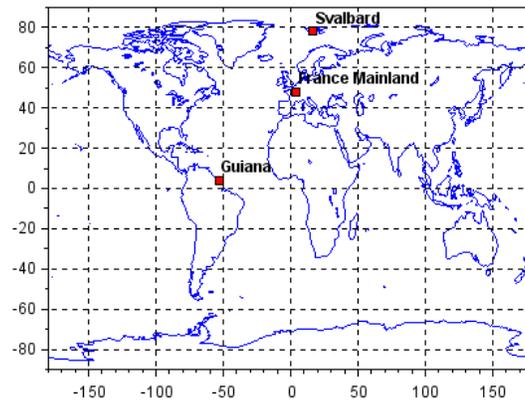


Fig. 2. Sensors locations

The simulation duration is seven days, each day the tracking sensors are tasked for the next 24 hours then the observations are generated assuming that each tasking is successful (to ease the analysis, constraints on the accuracy about the objects position so that measurements can be acquired by the tracking radar are not considered) and orbit determinations are performed using the measurements from both survey and tracking radar. A batch least square with an arc duration of 4 days is used and both the orbit and the object's drag coefficient are computed. At the end of the simulation, an object is considered as catalogued if the accuracy between the "real orbit", that is to say the orbit of the RSO, and the orbit of the catalog is under a given threshold (namely [40; 200; 100] m in QSW orbital frame) when propagated two days in the future.

The scheduler uses the following configurations:

- All objects are given the same priority (consistent with the fact that no manoeuvres are considered so all the objects are "debris")
- All sensors are given the same operation cost so no one is *a priori* preferred
- The "MIN_DISTANCE object heuristic" is used in order to track first the objects which are already close to meeting their accuracy requirement.
- 14 teams compete to find the best observation plan:
 - Half the team try to start a tracking as soon as possible within a visibility passage, while the other half try to track as late as possible
 - 12 teams use a random factor of 0.7 to select random observations whereas 2 teams do not apply *randomness*
 - The best observation plan is then selected as the plan which maximizes objects meeting their accuracy requirement with the lowest cost
- The expected covariance, that defines the accuracy constraint, is defined to be consistent with the cataloguing criterion used for the simulation: as a consequence, the scheduler goal is to increase the number of catalogued objects and not to increase the accuracy of the catalogue

3.2 Results

The reference performance is given by the network SNO with the surveillance radar alone: at the end of the simulation, 38% of the initial population is catalogued which represent 6800 objects in a full population. This figure, that can appear to be low, is the combination of three factors:

- The performance of the survey radar itself, which is able to observe only two thirds of the population (limitation to small size objects)
- The stringent cataloguing criterion (accuracy under a given threshold 48H after the end of the simulation)
- The simulation hypothesis, in particular the differences in the force model, that make it hard to catalogue an object even if it is observed frequently

Then, the main performance metrics that are analyzed are:

- The number of objects added in the catalogue
- The percentage of use of the tracking sensors during the last day of the simulation
- The number of tracked objects during the last day of the simulation
- The mean track length: averaged track duration for the object observed by the tracking sensor
- The mean number of daily tracks: averaged number of daily tracks generated by the tracking network

The percentage of use and number of tracked objects are analyzed on the last day of the simulation due to the fact that during the first days of simulation a transitory phase is observed because of the orbit determination arc having a length of 4 days.

Metric	SN1	SN2	SN3	SN4	SN5
Number of objects added in the catalog	9	159	137	181	362
% of use of tracking sensors (last day)	2.5 %	99.4 %	99.1 %	99.4 %	88.2 %
Number of tracked objects (last day)	23	382	373	350	632
Mean track length (s)	176	224	246	200	240
Mean number of daily tracks	121	388	355	432	939

Tab. 4. Performance metrics

For SN1, the percentage of use of the tracking sensors is very low: 2.5% over the last day of simulation. It is explained by the fact that the two low performance radars used within this network are able to observe only one third of the population due to RCS limitation. As a consequence, most of the objects that can be observed by those tracking radar are already well observed by the survey radar: the scheduler concludes that only a few objects are “worth tracking”. The added value of those low performance tracking radars is therefore null to increase the size of the catalog. However, with another setting of the scheduler (by increasing the target accuracy) one could estimate how they can contribute to increase the accuracy of the catalogue: this is identified as future work.

Other network (SN2, SN3, SN4 and SN5) only consider high performance tracking radars that are able to observe all the objects of the population.

The networks SN2, SN3 and SN4 consider one high performance radar each, placed in France mainland, Guiana and Svalbard respectively. Analyzing the results lead to the following considerations:

- In all those simulations, the scheduler tasks the tracking sensor at its maximum capacity (close to 100%) which is a sign of efficient scheduling (no down time)
- The number of objects added in the catalogue is maximum for Svalbard (181) and minimum for Guiana (137): while it is not surprising that locations at high latitudes are favourable, it is surprising to see that a site at opposite longitude from the survey radar (Guiana) does not show high performance. One explanation is that the scheduler tasks the tracking radar to observe in particular low inclined objects that still lack observations to be catalogued since they are not observed by the survey radar
- Between 350 and 382 object are tracked during the last day of simulation: the differences between the number of tracked objects and the number of objects added in the catalogue are linked to the difficulty to catalogue the objects (stringent cataloguing criterion, difference in the force model)
- Track lengths are between 200s and 224s: it means that the scheduler is not asking for observations during complete passes. It is particularly interesting to see that the number of daily tracks is maximized from Svalbard location: it is more efficient to observe objects several time during short periods – thus increasing the number of tracks – than to observe the object over a complete pass but less frequently (thus increasing the track length). This result was expected and is observed as an output of the scheduler.

The network SN5 consider three high performance radars in France mainland, Guiana and Svalbard: 632 objects are tracked during the last day of simulation and 362 are added in the catalogue: again, the difference is linked to the

difficulty to catalogue the objects (stringent cataloguing criterion, difference in the force model). The average percentage of use for the three tracking radars is around 90%. It is a consequence of two factors:

- The simulation is performed with a reduced population (10% of the full population) thus the number of objects that would require tracking is reduced as well
- One of the scheduler key setting is the accuracy constraint, defined through a target covariance: the scheduler tasks the tracking sensors to maximize the number of objects having a covariance smaller than the target one. The relationship between the covariance values and the fact that an object is catalogued or not is not straightforward since the cataloguing criterion is based on an accuracy between the RSO orbit and the catalogue orbit (not to mention the differences in the force model). As a consequence, the scheduler may assume that some objects will be catalogued, based on their covariance values, and decide not to track them while in reality their accuracy is not high enough for them to be catalogued.

The following plot analyse for the objects tracked during the last day of simulation which are the tracking sensor observing them:

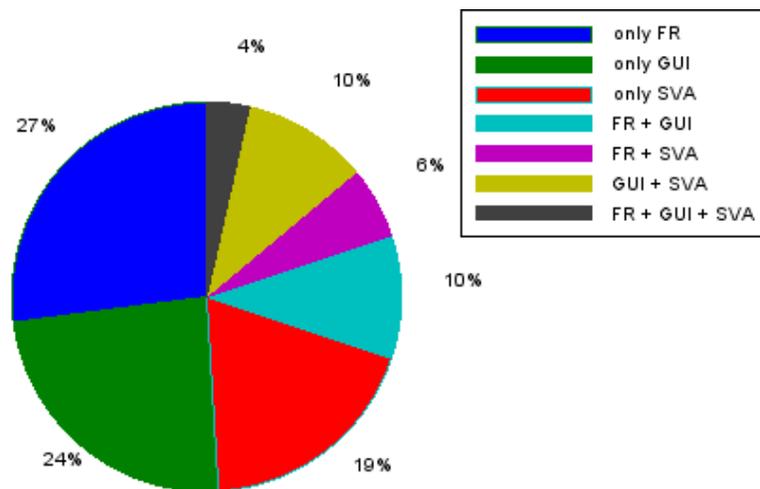


Fig. 3. SN5: percentage of the tracked objects seen by different sensors

It can be seen that:

- 70% of the tracked objects are observed by only one tracking sensor (27% from France mainland, 24% from Guiana and 19% from Svalbard)
- 26% of the tracked objects are observed by two tracking sensors, the less frequent combination being France mainland and Svalbard, which is consistent with the fact that those locations are near one another
- 4% of the tracked objects are observed by all three tracking sensors. It is worth noting that those objects are not observed at all by the survey sensor, thus justifying the use of all three tracking sensors
- All of this indicates that the co-ordinated tasking is efficiently performed by the scheduler

4 CONCLUSIONS

A complete simulation framework, BAS3E, has been developed at CNES. Applications cover the extensive testing of new data processing algorithm as well as the estimation of performance of current and future SST systems. In particular, a scheduler tool is available to solve the problem of coordinated tasking for tracking sensors, by optimizing the number of objects meeting a given accuracy criterion. Future work includes the use of such scheduler for optical sensors as well as the modeling of lasers within the simulation tool.

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6 REFERENCES

1. M. Boer et al. TAROT: a network for space surveillance and tracking operations. Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017
2. N. L. Johnson, P. H. Krisko, J. C. Liou, and P. D. Anz-Meador. Nasa's new breakup model of evolve 4.0. IEEE Trans. Aerosp. Electron. Syst., 28(9):1377-1384, 2001.
3. C.L. Stokely. Haystack and HAX radar measurements of the Orbital Debris Environment. NASA JSC-62815
4. S. Roussel, et al. Scheduling a network of sensors for Space Surveillance and Tracking: a CBLS approach. 10th *International Workshop on Planning and Scheduling for Space (IWSPSS)*, Pittsburgh, 2017
5. T. Flohrer, H. Krag & H. Klinkrad (2008). Assessment and Categorization of TLE Orbit Errors for the US SSN Catalogue. In Proc. Advanced Maui Optical and Space Surveillance Technologies Conference, 17 - 19 September 2008, Wailea, Maui, USA.
6. Vallado, D. A. *Fundamentals of astrodynamics and applications. Third edition.* Springer. 2007
7. Gooding, R. H. A new procedure for the solution of the classical problem of minimal orbit determination from three lines of sight. *Celestial Mechanics and Dynamical Astronomy*, 66(4), 387-423. 1996
8. Gronchi, G. F., Bau, G., & Maro, S. Orbit determination with the two-body integrals: III. *Celestial Mechanics and Dynamical Astronomy*, 123(2), 105-122. 2015
9. Siminski, J. A., Montenbruck, O., Fiedler, H., & Schildknecht, T. Short-arc tracklet association for geostationary objects. *Advances in space research*, 53(8), 1184-1194. 2014
10. C. Yanez, F. Mercier, and J-C. Dolado. A novel initial orbit determination algorithm from Doppler and angular observations. Proc. 7th European Conference on Space Debris, Darmstadt, Germany, 18–21 April 2017
11. Maruskin, J. M., Scheeres, D. J., & Alfriend, K. T. Correlation of optical observations of objects in earth orbit. *Journal of Guidance, Control, and Dynamics*, 32(1), 194-209. 2009
12. Serra, R., Arzelier, D., Joldes, M., Lasserre, J. B., Rondepierre, A., & Salvy, B. Fast and accurate computation of orbital collision probability for short-term encounters. *Journal of Guidance, Control, and Dynamics*, 1009-1021. 2016
13. Tapley, B., Schutz, B., & Born, G. H. *Statistical orbit determination.* Elsevier. 2004