Small Debris Estimation Approach Using Sparse Sampling to Infer Markov Steady State Distributions

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ABSTRACT

Space systems are a vital part of the global information-sharing infrastructure. The strategic utility of space as an operating environment enables services such as precision navigation and timing, weather forecasting, and communications. While space is becoming increasingly utilized by a growing number of space-faring nations and organizations, the environment continues to become increasingly cluttered with debris particles.

Large debris particles can be tracked using space surveillance sensors, and well-established algorithms can predict conjunctions with enough lead time for a satellite operator to employ a countermeasure. Very small debris particles (such as nanoparticles), cannot be detected by space surveillance sensors, but have sufficiently low mass that they do not pose a practical threat to operational satellites. In between these two classes of particles, there exists a range of debris, in size and mass, which are too small to be detected by space surveillance sensors, but still pose a threat to satellite operations. We refer to debris particles that lie in this observability-risk class (nominally 10^{-4} to 10^{-1} meters in apparent diameter) as minimally observable destructive debris (MODD) particles.

While active tracking of MODD particles is a desired outcome, the physical and economic limitations associated with sensor phenomenologies of conventional space surveillance sensors, such as the diffraction limit of electro-optical sensors, make this a practical impossibility. Instead, a more reasonable objective is to characterize the spatial density of MODD particles and build probabilistic distributions. It may be possible to collect data on the presence of MODD particles using impact plates which are launched into various orbits for the purpose of collecting impact signatures. Then the challenge lies in how to characterize the distribution of MODD particles, on the basis of sparse observable data from collection methods such as impact plates. Therefore, the research question that we pose is: What is the optimal data collection policy to maximally recover the true distribution of MODD particles, given sparse and irregular data collection opportunities?

The approach that we use is to frame the distribution of MODD particles as a discrete state space, finite time horizon Markov process. Given this framework, we attempt to recover estimates of the true steady state distribution from the sparsely sampled data. From the observable data, we construct an estimated Markov process and use detailed balance conditions to estimate the true steady state distribution of MODD particles. The objective is to find an optimal data collection policy to help inform mission requirements for such a data collection campaign.

1 INTRODUCTION

Space systems provide critical services that benefit, either directly or indirectly, virtually every living person. These systems provide services such as secure communications, precision navigation and timing, and weather monitoring, among many others. With advances in the miniaturization of space technologies and decreasing launch costs, space has become increasingly accessible to space-faring nations and organizations. While large nation states continue their exploitation of space as a staging environment for GPS, Galileo, and other constellations comprised of large satellites, emergent space-faring organizations such as OneWeb [1] and SpaceX Starlink [2] are on the cusp of inserting massive constellations of hundreds or thousands of satellites.

These space systems face the risk of unintended collision with other satellites, both active and inactive, and with debris. Conjunctions with even microscopically small debris particles can result in catastrophic damage to space systems. For these reasons, several networks of space surveillance sensors operate to provide space situational awareness. Networks such as the Combined Space Operations Center (CSpOC) and the International Scientific Optical Network (ISON) provide observations of the space environment to maintain catalogs of tracked resident space objects. The CSpOC, in particular, maintains the authoritative catalog of resident space objects, comprised of approximately

25,000 satellites and debris particles in near Earth and deep space [3]. This catalog is updated using a set of optical and radar sensors called the United States Space Surveillance Network. These sensors are capable of tracking resident space objects in the geosynchronous Earth orbit that are larger than approximately 10 centimeters in apparent diameter at low Earth orbit, and approximately 1 meters in apparent diameter at the geosynchronous Earth orbit [4].

Given the current sensing capabilities by the various networks of space surveillance sensors, there exists three classes of debris. Very small debris particles, which are smaller than approximately 10^{-4} meters in apparent diameter, are likely to not inflict appreciable damage to space systems due to their low mass. Large debris particles, which we define as particles that are greater than 10^{-1} meters in apparent diameter, are likely to cause catastrophic damage to space systems [5]. However, the number of these particles are relatively small compared to the total population of orbiting particles, and also can be tracked by the existing networks of space surveillance systems. Between these size classes, we refer to a population of debris particles called minimally observable destructive debris (MODD). These debris, nominally ranging from 10^{-4} meters to 10^{-1} meters in apparent diameter, are smaller than the minimum observable size threshold for most space surveillance sensors, and yet are large enough to inflict severe or catastrophic damage to space systems.

MODD debris particles pose a uniquely challenging problem: while they are too small to reliably detect and track, they are large enough to cause significant damage to space systems. One method of addressing this challenge is to build better optical and radar systems. However, it is difficult to imagine that the diffraction limit constraints of optical systems can be overcome to reliably track MODD particles. Furthermore, the power requirements for radars, when detecting the large number of MODD particles, likely make this an intractable solution for tracking a significant number of MODD particles. Therefore, we claim that the only plausible method of managing the population of MODD particles is through probabilistic characterization of debris densities, rather than individual particle tracking.

Given this claim, we pose the following research questions:

(1) How can the stochasticity associated with MODD particle density be characterized?
(2) Can the stochasticity of MODD particle density be used to predict future density distributions?
(3) What is the optimal sampling pattern to optimally characterize MODD particle density distributions with sparse sampling?

The following sections discuss the analysis performed to answer these questions.

2 BACKGROUND

Space debris continues to threaten the safe operations of satellite constellations. In their most recent Orbital Debris Quarterly News, NASA reports a generally increasing trend in the number of total cataloged objects and fragmentation debris, over time [6]. While NASA's report depicts the growing number of cataloged objects, generally comprised of particles larger than 10 centimeters in apparent diameter, the number of MODD particles is significantly higher. In a 2010 report by the Union of Concerned Scientists, they estimated that there were 24,000 debris particles in all orbits greater than 10 centimeters in apparent diameter, but also 750,000 debris particles between 1 centimeter and 10 centimeters in apparent diameter [5]. One can reasonably infer that as the number of cataloged debris particles increases over time, the number of non-cataloged MODD particles has also increased, possibly at a faster rate.

The destructive potential of debris particles is significant and well documented. In a 2012 report, Christiansen and Lear provided a background on micro-meteoroid and orbital debris (MMOD) particles. Their report provides examples of actual known damage to the International Space Station (ISS) and the Space Shuttle from impacts with MMOD particles greater than 1 millimeter. Some examples include ISS compressor damage due to a 2 millimeter – 3 millimeter diameter particle, ISS radiator damage due to a 3 millimeter – 5 millimeter particle, and damage to the Shuttle's radiator due to a 1 millimeter particle [7]. The European Space Agency provides some heuristics on hypervelocity impact effects. They state that for projectiles with velocities greater than 4 km/sec, "an impact will lead to a complete break-up and melting of the projectile, and an ejection of crater material to a depth of typically 2–5 times the diameter of the projectile" [8].

Space debris originate from several different classes of sources, and given these source types and their varying origination phenomenologies, there is a wide range of debris particle sizes and velocities. The study of debris particles has been given considerable attention in the recent years, and the literature is extensive. In our model, we present a slightly different treatment of small debris, by proposing a method of characterizing debris density of MODD particles as a Markov process.

3 MATHEMATICAL MODEL

We approach the characterization of MODD particles as a stochastic process. In particular, we create a discrete state space Markov framework to describe the stochasticity of MODD particles and predict future distributions. This framework is used because of the computational convenience associated with Markov chains.

In this section, we describe the mathematical model used in our analysis. First, we start with a description of our primary data source, the European Space Agency (ESA) Meteoroid and Space Debris Terrestrial Environment Reference (MASTER) tool suite. Next, we discuss the Markov Property and define the state space and Markov state transition matrix. Then, we describe the method by which future state distributions are computed. Finally, we describe two plausible methods of simulating sparse sampling patterns of the Markov steady state in order to recover the true steady state distribution.

3.1 ESA MASTER Model as Truth Proxy

The MASTER tool suite allows users to generate debris and meteoroid flux maps on various Earth orbits up to the geosynchronous Earth orbit. The underlying models used by MASTER are quasi-deterministic physics-based models, using orbit propagation techniques and volume discretization. The output of MASTER simulations are debris spatial density maps by altitude, declination, and right ascension, thus creating discretized spatial density boxes [10].

We use the MASTER-generated spatial density maps as a proxy for the true time-phased spatial densities of MODD particles in the orbits of interest, namely the geosynchronous Earth orbit. This is a necessary step because the true MODD particle densities are an unobservable stochastic process. We believe the use of MASTER is a justifiable approach for two reasons. First, the ESA's MASTER model has been published in peer-reviewed journals extensively, including in several high impact publications such as Advances in Space Research [12], [13] and Acta Astronautica [14]. We interpret this publication record as a form of model validation that is sufficient for this study. Second, the MASTER model serves as a sufficient starting point, from which probabilistic updating can occur using Bayes' Theorem. Since we are creating a debris framework based on probability distributions, we can interpret the MASTER data as a prior belief state and use new information, in the form of in-situ measurements or updated physics-based models, to update those priors into posterior probability distributions. The method by which these conditional posterior probabilities are derived given newly observed data is the well-known process of Bayesian updating [15]. It should be noted that data from other debris models, such as NASA's Orbital Debris Engineering Model (ORDEM), could be used as the primary data source as a follow-on research activity.

3.2 Discrete State Space Markov Model

The key foundational concept in our model is the presumption that MODD particle densities obey the Markov Property, which states that in a stochastic process, the distribution of the next state is known in the current state, and no additional information is gained if the entire past history of the process is known. Mathematically, let $X_0, X_1, \ldots$ be a sequence of states. Then

$$P(X_{t+1} = x_{t+1} | X_t = x_t) = P(X_{t+1} = x_{t+1} | X_0 = x_0, \ldots, X_t = x_t) \tag{1}$$

In our model, the discrete state space is comprised of the spatial density of MODD particles for each volumetric box in space ($X^{[B]}$) along with the average spatial density of all neighboring volumetric boxes ($X^{[N]}$), collectively represented as the random variable X. We flatten the state transitions to a two-dimensional transition space by creating superstates comprised of both the spatial density of a specific volumetric box, and the average spatial density of its neighboring volumetric boxes. Therefore, the size of the state space is

$$|X| = \left|X^{[B]}\right| \times \left|X^{[N]}\right| \tag{2}$$

In our study, we discretize spatial densities (for both the current box and the neighboring boxes) into 100 possible values, therefore, the state space $x \in X$ is the following:

$$
\begin{aligned}
x_1 &= \left(x_1^{[B]}, x_1^{[N]}\right) \\
x_2 &= \left(x_1^{[B]}, x_2^{[N]}\right) \\
&\vdots \\
x_{100} &= \left(x_1^{[B]}, x_{100}^{[N]}\right) \\
x_{101} &= \left(x_2^{[B]}, x_1^{[N]}\right) \\
x_{102} &= \left(x_2^{[B]}, x_2^{[N]}\right) \\
&\vdots \\
x_{200} &= \left(x_2^{[B]}, x_{100}^{[N]}\right) \\
&\vdots \\
x_{10000} &= \left(x_{100}^{[B]}, x_{100}^{[N]}\right)
\end{aligned}
\tag{3}
$$

The resulting Markov chain is a discrete state space Markov chain with a single, closed, irreducible communicating class with the following structure:

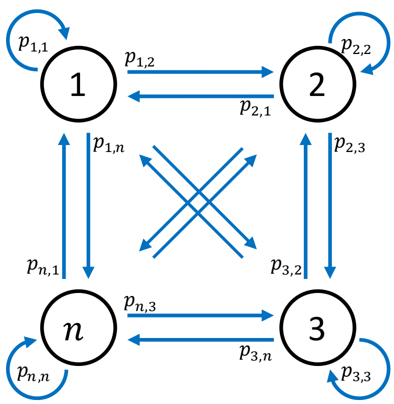

Figure 1. Sample discrete state space Markov chain.

From this formulation of the Markov chain, the objective is to compute the steady state distribution for a finite time horizon. We next describe the computation of the steady state distribution.

3.3 Markov Steady State Distribution

The state transition matrix is fully spanning, and takes the following form:

$$
\Pi = \begin{bmatrix}
P(x_1|x_1) & P(x_2|x_1) & \cdots & P(x_i|x_1) & \cdots & P(x_{10000}|x_1) \\
P(x_1|x_2) & P(x_2|x_2) & \cdots & P(x_i|x_2) & \cdots & P(x_{10000}|x_2) \\
\vdots & \vdots & & \vdots & & \vdots \\
P(x_1|x_j) & P(x_2|x_j) & \cdots & \boldsymbol{P(x_i|x_j)} & \cdots & P(x_{10000}|x_j) \\
\vdots & \vdots & & \vdots & & \vdots \\
P(x_1|x_{10000}) & P(x_2|x_{10000}) & \cdots & P(x_i|x_{10000}) & \cdots & P(x_{10000}|x_{10000})
\end{bmatrix}
\tag{4}
$$

where $\sum_{i=1}^{10000} P\left(x_i|x_j\right) = 1, j = 1,2, \dots, 10000$ are probability distributions.

Given this stochastic matrix for a discrete state space Markov chain, it is possible to derive the steady state distribution using the following detailed balance conditions:

$$\Pi(i)P\left(x_i, x_j\right) = \Pi(j)P\left(x_j, x_i\right) \tag{5}$$

for all x_i and x_j, where x_i and x_j are pairs of communicating states. Any distribution $\Pi(i)P\left(x_i, x_j\right)$ that satisfies these detailed balance conditions is the invariant steady state distribution $P(x_{SS})$. The detailed balance conditions can be used to create a system of equations that finds $P(x_{SS})$:

$$\begin{gathered}
\Pi(1)P(x_1, x_2) = \Pi(2)P(x_2, x_1) \\
\Pi(1)P(x_1, x_3) = \Pi(3)P(x_3, x_1) \\
\vdots \\
\Pi(1)P(x_1, x_{10000}) = \Pi(10000)P(x_{10000}, x_1) \\
\Pi(2)P(x_2, x_3) = \Pi(3)P(x_3, x_2) \\
\Pi(2)P(x_2, x_4) = \Pi(4)P(x_4, x_2) \\
\vdots \\
\Pi(9999)P(x_{9999}, x_{10000}) = \Pi(10000)P(x_{10000}, x_{9999})
\end{gathered} \tag{6}$$

While detailed balance conditions produce exact steady state distributions, they are can be computationally intractable for large state spaces. However, given the formulation of the stochastic process as a discrete state space, closed, irreducible communicating class Markov chain, there exists a key characteristic that enables relatively easy computation of steady state distribution approximations.

Lemma: Suppose X_t is a discrete state space, irreducible stochastic process that obeys the Markov Property with state transition matrix Π. Then $P(x_t) = P(x_0) \times \Pi^t$.

Suppose $P(x_0) = P(X_0 = x_0)$. Then:

$$\begin{aligned}
&P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}, X_t = x_t) \\
&\qquad = P(X_t = x_t|X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}) \times P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1})
\end{aligned}$$

by the rule of joint probabilities. Then by invoking the Markov Property, we have

$$\begin{aligned}
&P(X_t = x_t|X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}) \times P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}) \\
&\qquad = P(X_t = x_t|X_{t-1} = x_{t-1}) \times P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1})
\end{aligned}$$

By invoking the rule of joint probabilities again, we have

$$\begin{aligned}
&P(X_t = x_t|X_{t-1} = x_{t-1}) \times P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}) \\
&\qquad = P(X_t = x_t|X_{t-1} = x_{t-1}) \times P(X_{t-1} = x_{t-1}|X_0 = x_0, X_1 = x_1, \dots, X_{t-2} = x_{t-2}) \\
&\qquad \times P(X_0 = x_0, X_1 = x_1, \dots, X_{t-2} = x_{t-2})
\end{aligned}$$

This process is repeated until all joint probabilities are separated according to

$$P(X_0 = x_0, X_1 = x_1, \dots, X_{t-1} = x_{t-1}, X_t = x_t) = P(X_t = x_t|X_{t-1} = x_{t-1}) \times P(X_{t-1} = x_{t-1}|X_{t-2} = x_{t-2}) \cdots P(x_0)$$

Assuming time-homogeneous transitions, where

$$P(X_t = x_t|X_{t-1} = x_{t-1}) = P(X_{t-1} = x_{t-1}|X_{t-2} = x_{t-2}), \forall t$$

then we can rewrite this expression as

$$P(X_t = x_t) = P(X_t = x_t|X_{t-1} = x_{t-1})^t \times P(x_0)$$

Then we can represent all probabilistic state transitions as $\Pi = P(X_t = x_t|X_{t-1} = x_{t-1})$ to get

$$P(X_t = x_t) = P(X_t = x_0) \times \Pi^t \tag{7}$$

This result is important because we can easily compute the steady state distribution for the Markov chain by computing Equation (7) for t large as an alternate to the more computationally intense process of using detailed balance conditions.

3.4 Simulated Sparse Sampling and Markov Steady State Distribution Recovery

For t large, $P(X_t = x_t) = P(X_t = x_0) \times \Pi^t$ computes the steady state distribution for irreducible Markov chains. Then the objective is to find $\hat{P}(x_t)$ that optimally recovers the true $P(x_t)$, given sparse sampling of the underlying stochastic process, which are changes in the MODD particle spatial density for a given volumetric box in space. The algorithm for computing $\hat{P}(x_t)$ is the following:

- Sample state transitions, tabulate them into Π_{count}
- For $n \in (1, \ldots, N)$ samples
  - For each row vector in Π_{count}
    - Divide each cell by total transitions to create a probability distribution
    - Output: probability transition matrix $\hat{\Pi}$
  - Given a diffuse initial state x_0, compute $\hat{P}(x_t) = P(x_0) \times \hat{\Pi}^t$ for t large
    - $\hat{P}(x_t)$ is the estimated steady-state debris density distribution, given $\hat{\Pi}$
  - Compute root mean square error (RMSE) between $\hat{P}(x_t)$ and true $P(x_t)$ for all t

While a more exhaustive study of optimal sampling patterns is planned for further study, in this effort we explore two sample patterns: (1) uniform random sampling of all states in the state space, and (2) tailored sampling of states for states with nonzero probabilities in the original steady state distribution $P(x_t)$.

In the following section, we describe the application of our model to the geosynchronous orbit and provide results.

4 RESULTS OF MARKOV MODEL APPLIED TO MODD PARTICLE SPATIAL DENSITY

Our analysis follows the workflow depicted in Figure 2.

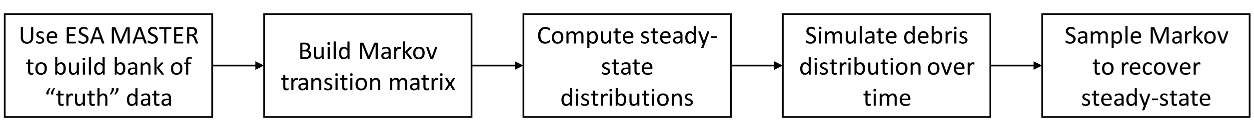

Figure 2. Markov study roadmap.

4.1 Markov Projection of Future MODD Particle Spatial Densities

We begin our analysis with the ESA MASTER tool suite of spatial densities for particles ranging 10^{-4} to 10^{-1} meters in apparent diameter. MASTER is used to generate a temporal datacube of particle spatial densities given the following constraints: Alt $\in (35000{,}36500)$ kilometers, Dec $\in (-90{,}90)$ degrees, and RA $\in (-180{,}180)$ degrees. This total volume is discretized into individual volumetric boxes, within which each box is characterized as a discrete state space Markov chain. For $t = 1, \ldots, 200$ timesteps in units of one month (from January 2000 to August 2016) we compute spatial densities for each discretized volumetric box in the geosynchronous orbit. From timestep-to-timestep, we tabulate state transitions $(X_t|X_{t-1})$ and normalize to probability distributions to derive the state transition matrix Π.

This state transition matrix Π is then used to project future distributions, starting at t_0, and the Markov-based projections are compared to the ESA MASTER truth for the same simulation period used to derive the state transition matrix Π. A sample of the resulting projections are depicted in Figure 3.

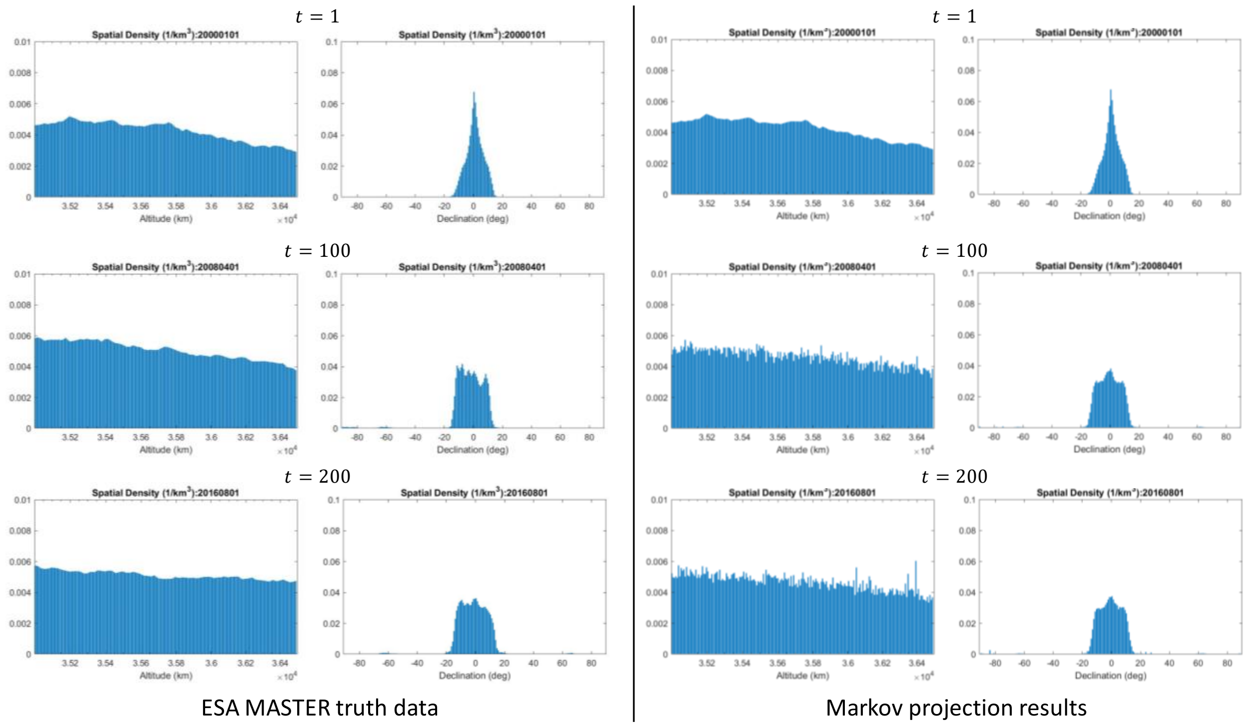

Figure 3. ESA MASTER truth data for spatial densities at GEO with respect to altitude and declination (left), Markov projection results of spatial densities at GEO with respect to altitude and declination (right).

For each timestep t, the RMSE with respect to the ESA MASTER truth MODD particle density distributions are computed. Over 200 timesteps, the resulting RMSE values are depicted in Figure 4. With respect to declination, $RMSE_{mean} \approx 6.5 \times 10^{-3}$ and with respect to right ascension, $RMSE_{mean} \approx 5.5 \times 10^{-4}$.

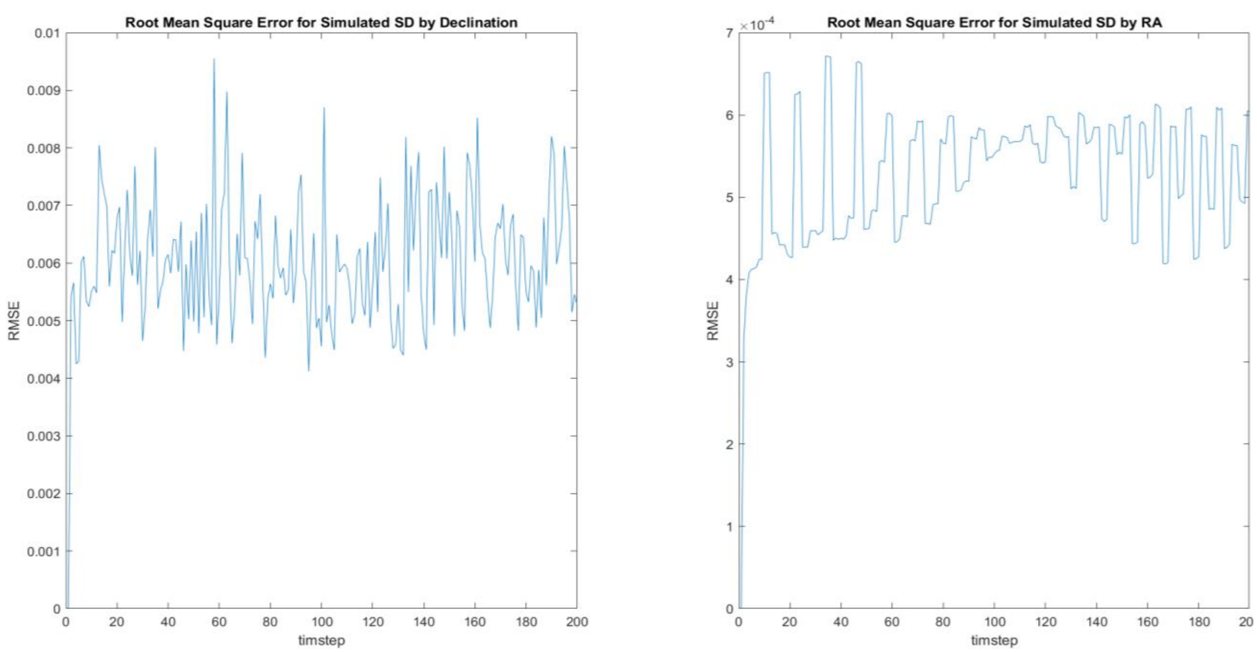

Figure 4. RMSE of Markov projection results of spatial densities at GEO with respect to declination (left) and right ascension (right).

4.2 Optimal Sparse Sampling to Recover Markov Steady State

Lastly, given the computed steady state Markov distribution for each volumetric box in the geosynchronous Earth orbit computed by $P(X_t = x_t) = P(X_t = x_0) \times \Pi^t$ for t large, which we denote as $P(x_{SS})$, we are interested in finding an optimal sampling pattern that maximally recovers the distribution given sparse and possibly irregular data sampling. Such a scenario may be possible with in-situ sensors at the geosynchronous Earth orbits using impact plates, which can report changes in MODD particle spatial density over time.

While more extensive study will be performed on this topic in a future research effort, in this study effort we explore two sparse sampling methods to recover $P(x_{SS})$: (1) uniform random sampling across the full state space, (2) tailored sampling for non-zero portions of the state space from the $P(x_{SS})$ distribution. Given the algorithm described in Section 3.4, we apply both methods and compute RMSE over multiple simulated samples with the following results:

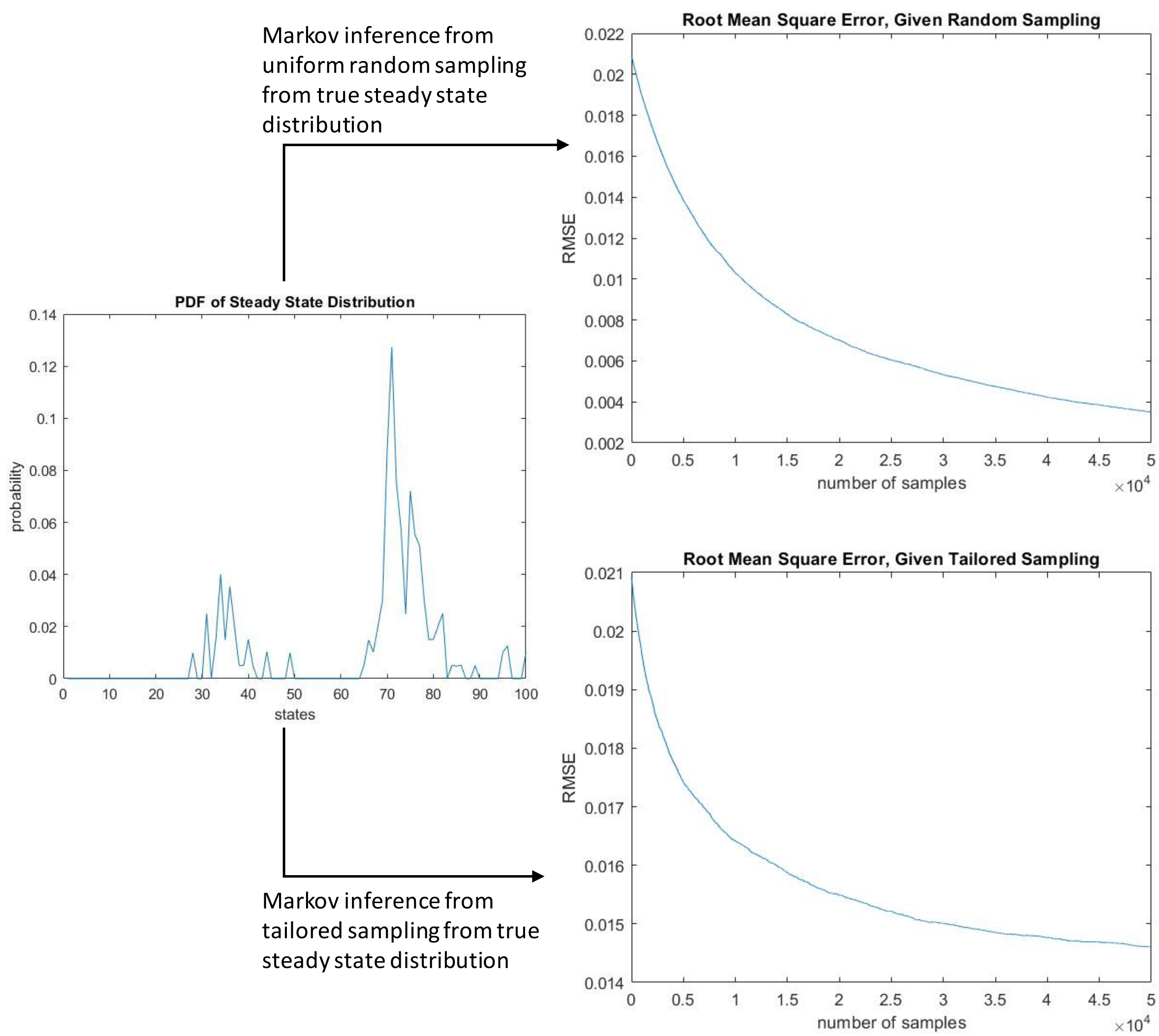

Figure 5. Sample PDF of a steady state Markov distribution (left). RMSE from steady state distribution estimation from random sampling (top right) and from tailored sampling (bottom right).

The results of this analysis demonstrate that tailored sampling of the steady state distribution leads to poorer recovery of the true steady state distribution, compared to uniform random sampling. Our analysis suggests that the lower RMSE results with respect to the true steady state distribution when sampling all states, even null states in the true steady state distribution. Therefore, we conclude that uniform random sampling across all states in the state space is the best method of recovering the true steady state distribution.

5 CONCLUSION

In this research effort, we address the problem of managing minimally observable destructive debris (MODD) particles in space. In particular, we are interested in characterizing the changing spatial densities of MODD particles in the geosynchronous Earth orbit. While efforts abound in attempting to track individual particles, we claim that even significant advancements in sensing technologies will not enable effective particle tracking of small debris particles. Therefore, we propose a method using discrete state space Markov chains to characterize the steady state probability distributions of MODD particle spatial densities. By using the European Space Agency's MASTER tool suite as a proxy for truth data, we train our model and build a state transition matrix based on a state space that is comprised of spatial densities in a volume of interest and its neighboring volumes. This model is used to project future spatial densities with low RMSE compared to the truth data. Moreover, we demonstrate two hypothetical data collection campaigns to recover the true steady state distribution, given sparse sampling. Among these, pure random sampling of all states in the state space is the best method. There are several opportunities for further study. These include further analysis of optimal sparse sampling to recover the true steady state distribution, and analysis of additional Earth orbits, such as the low Earth orbit and medium Earth orbit.

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