

# DEVELOPMENT OF THEORY OF PHYSICAL LIBRATION OF THE MOON TAKING INTO ACCOUNT THE LUNAR TWO-LAYER MODEL INCLUDING A SOLID MANTLE AND A LIQUID CORE

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**Introduction:** The problem of including a model of a two-layer Moon (with a small liquid core inside) into the theory of rotation became relevant after the effects of its dissipation at lunar rotation were discovered. These effects arise both due to tidal friction and due to turbulent processes at the boundary between the liquid core and the solid mantle. Williams et al. [1], based on computer simulations of long-term observations of laser data, managed not only to estimate the size of the liquid core, but also to describe its chemical composition, including the inner solid core. In addition to dissipation, the presence of the core in the theory of lunar rotation also manifests itself in the form of free core nutations (FCNs). Therefore, the inclusion of the effects of differential rotation of the core and mantle into the theory of lunar libration, on the one hand, will make it possible to detect new modes in the expansion of the physical libration of the Moon (PLM) in observations and to estimate independently the characteristics of the core; on the other hand, the inclusion of these effects will allow to increase the accuracy of the developed theory to the required level

**Methods:** In order to obtain refined values of the parameters of PLM theory, taking into account the two-layer model of the Moon, which includes a solid mantle and a liquid core, an algorithm was developed for determining the PLM parameters at the mathematical and program levels. The two-layer model of the Moon is a solid body with an ellipsoidal cavity filled with an incompressible fluid. This is called the Poincaré problem. The analytical theory [2] includes a similar model [3]. This model has a moment of inertia from the liquid core of the Moon in relation to the entire core (liquid core and solid inner core) of 71.3%. This allows, in the first approximation, to consider a two-layer model in which the core is completely liquid. To describe the rotation of a biaxial rigid body with a biaxial ellipsoidal cavity filled with liquid, we constructed the Euler-Liouville equations, which were reduced to the canonical form of the Hamilton equations. To solve them, we developed the necessary software.

**Results:** Based on our theory taking into account the liquid core, a powerful analytical tool was obtained for modeling PLM observations to determine the frequencies – “markers” (FMs) in libration series whose amplitudes are most sensitive to the presence of a liquid core. At the same time, in order to determine the maximum possible number of such harmonics, one can simulate the parameters of the core with hypothetically overestimated values of the radius and flattening of the core. The empirical PLM series [4] were used as the observed data for comparing the dynamic ephemeris DE421 with long-term laser observations [5–10]. Most of the FMs coincide with the terms of the semi-empirical series [4], whose nature is unknown.

**Conclusions:** The creation of PLM theory for the model with a liquid core has provided the necessary tool for detecting harmonics with low amplitude and long periods in observations, which allow one to study hidden information about the parameters of the lunar core. Based on our theory and using computer simulations, we managed to effectively analyze the lunar rotation in order to refine the internal structure of the Moon [11–16].

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