DYNAMIC COORDINATES OF MÖSTING A LUNAR CRATER

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Introduction: Knowledge of the rectangular dynamic coordinates of Mösting A crater is important in studying the rotation and figure of the Moon [1]. The data for determining the coordinates of this crater can be heliometric observations of the crater relative to the stars, which, as it is known, sufficiently differ in accuracy for this purpose [2]. However, studies show that some heliometric measurements are anomalous [3]. Therefore, estimation of the crater coordinates using the classical least squares method (LSM) requires the elimination of erroneous measurements (cutting), since the latter have a strong influence on the LSM estimates. At the same time, the concern is quite justified (this finds its theoretical and practical confirmation) that it is impossible to reliably detect rough measurements, since there may be erroneous preservations.

Methods: A new approach to the problem of parameter estimation provides the use of modern methods of processing observations – robust statistical procedures (SP) [4]. The main purpose of SP is to eliminate the influence of errors of anomalous observations [5]. As source material in this work, we used the conditional equations of connection between the calculated dynamic coordinates of Mösting A and the observed positions of the crater, obtained by linking it to the stars on the heliometer [6]. The M-estimation method was used to determine the robust values of the dynamic coordinates of Mösting A crater. When constructing M-estimates of coordinates, the Huber function was used. When calculating, 2 options were considered: 1) M1 – estimates found under the assumption of a normal distribution of measurement errors; 2) M2 – estimates produced with 95% efficiency.

Results: An analysis of the results that were produced by two alternative statistical procedures showed that: 1) the cut sample may contain anomalous measurements; 2) the use of a robust procedure for estimating dynamic coordinates is appropriate and justified; 3) as estimates of the dynamic coordinates of Mösting A crater, one should use M-estimates $\xi = -0.09112 \pm 2\times10^{-5}$, $\eta = -0.05588 \pm 2\times10^{-5}$, $\varsigma = 0.99417 \pm 10\times10^{-5}$. Here ξ is a rectangular selenographic axis directed along the axis of rotation of the Moon, η is a rectangular selenographic axis directed to the west, ς is an axis directed to the Earth. All coordinates are measured from the center of mass of the Moon and coincide with its axes of inertia. The values of the coordinates are given in fractions of the mean radius of the Moon. As can be seen from the obtained parameters, the coordinate directed to the Earth has a lower accuracy compared to the other two. This is explained by the fact that this coordinate is derived from the stereo effect due to optical librations.

Conclusions: Comparison of LSM-estimates and M-estimates of the coordinates of Mösting A crater made it possible to determine anomalous measurements in the sample of observations that were accepted for processing. The method developed in the work can be used to create regression models both for calculating the coordinates of the desired objects on the lunar surface [7–9], and for analyzing other selenodetic processes [10–12] and the planetary physical properties of the Moon.

Acknowledgements: This work was partially supported by Russian Science Foundation, grants no. 20-12-00105 (according to the grant, the method for data analysis was created and the numerical calculations were carried out). This work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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