CHARACTERIZING MAGNETIC FORCE BETWEEN PARAMAGNETIC PARTICLES OF MAGNETIC ASTEROIDS. A. Sikka*[1], I. DesJardin[1] and, C. Hartzell[1], [1]Aerospace Engineering, University of Maryland, College Park, MD (*sikka@umd.edu)

Introduction: Magnetic fields have been detected at asteroids Gaspra [1] and Braille (2 nT) [2]. While these are S-type asteroids and the detections are weakened by other factors, 5% of the main-belt is comprised of M-type asteroids that may have significant magnetic fields. These asteroids, constituted primarily of Nickel and Iron, are thought to be exposed planetesimal cores [3]. A prehistoric dynamo could have caused a magnetic field to be locked into the cooling material. Scheinberg et al. [3] have studied the evolution of the dynamo during solidification, particularly in the context of the asteroid Psyche. The Psyche mission will carry a magnetometer to its eponymous asteroid. On-going simulation work has modeled the plasma environment around Psyche with magnetic fields ranging from 0-500nT [4]. Additionally, it is natural to assume that metallic asteroids will also be subject to the disruption and re-accretion processes that is thought to produce rubble pile asteroids from other taxonomic classes. Thus, metallic asteroids may exist in various sizes and magnetic field strengths.

Previous work by Scheeres et al. [5] showed that the van der Waals cohesion is a significant force between grains on traditional (non-metallic) asteroids. Subsequently, Sanchez and Scheeres [6, 7] worked on the influence of the cohesive force towards the evolution of rubble-pile asteroids. The surface morphology of the regolith is influenced by the dominant force [8].

The regolith on M-type asteroids with embedded magnetic fields will experience another cohesion-like force due to the magnetic forces between particles. This force may be attractive or repulsive. Since cohesion has been hypothesized to significantly influence the surface morphology of asteroids, we hypothesize that magnetic forces will similarly influence the morphology of metallic asteroids with embedded magnetic fields. We present results showing the size of particles where the magnetic attraction between particles is equal to the weight of a particle, for varying gravity, magnetic field strength and magnetic susceptibility. We also present an empirical fit that enables users to quickly compute the magnetic force between identical particles, reducing the need to implement complicated and computationally costly magnetic multipole models.

Magnetostatic Problem: Paramagnetic particles placed in a uniform magnetic field develop an induced dipole moment. The differential equations for the magnetic interactions between particles involve the Laplace equation. By applying appropriate boundary conditions, a solution can be generated.

Laplace's equations for a system of N paramagnetic spherical particles in a uniform magnetic field $\mathbf{H}_0$ without any free currents are [9]

$$\nabla^2\phi_{out} = 0 \tag{1}$$

$$\nabla^2\phi_{in}^{(n)} = 0 \tag{2}$$

where ϕ_{out} is the scalar magnetic potential outside the particles and ϕ_{in} being inside the particles. For a spherical co-ordinate system defined at the center of nth sphere such that $\mathbf{r}_n = \mathbf{x} - \mathbf{X}_n$ with $r_n = |\mathbf{r}_n|$, the boundary conditions are

$$\phi_{in}^{(n)} = \phi_{out} \tag{3}$$

$$\mu\frac{\partial\phi_{in}^{(n)}}{\partial r_n} = \mu_0\frac{\partial\phi_{out}}{\partial r_n} \tag{4}$$

at $r_n = a$ for each $n = 1, 2$ where r_n is radial coordinate, a is particle radius, μ_0 is the free space permeability and μ is the particle permeability. Additionally, as $r_n \to \infty$

$$\phi_{out} \to \mathbf{H}_0 \cdot \mathbf{r}_n \tag{5}$$

The general solution to Laplace's equation for a spherical geometry is the sum of spherical harmonics. Four different models are considered to solve for the magnetic force between particles.

Fixed Dipole Model (DM): Consider a single spherical particle in a uniform magnetic field; the particle is uniformly magnetized. In this model, only this magnetization of a single particle in a uniform magnetic field is considered. The interaction terms between the particles are neglected.

Mutual Dipole Mode (MDM): In this model, on top of the magnetization due to the external magnetic field, the magnetic field due to all other magnetized particles is also considered. The dipole moments of each particle are iterated until they converge.

While the MDM solves the interaction between the particles, the effect of higher-order multipole terms is still not included. Both DM and MDM diverge from the exact solution, particularly at low separation distances. To include the higher-order multipole terms, we need to solve Laplace's equations.

Finite Volume Method (FVM): Laplace's equations with the applied boundary conditions can be numerically solved through a finite volume method [10] using a smoothed μ approximation. This method exactly solves for the magnetic field without neglecting harmonic terms. However, it is computationally expensive and was only used to benchmark simpler methods.

Spherical Harmonics Solver: The models were compared for a two-body magnetostatic problem. A spherical harmonics solver [11] is used to obtain the exact solution. By terminating the sum of spherical harmonics up to finite multipoles 'L' and applying the Hobson formula, we arrive at a $2L \times 2L$ linear system of equations. The solution to the linear system provides us the scalar potential ϕ_{out} and therefore, the contribution of the magnetic field from the particles by taking the gradient. The force between the particles is then computed using Maxwell stress tensor:

$$\mathbf{T} = \mu_0 \Big(\mathbf{HH} - \frac{1}{2} H^2 \mathbf{I} \Big) \tag{6}$$

$$\mathbf{F}_n = \int_{V_i} \nabla \cdot \mathbf{T} \, dV \tag{7}$$

where F_n is the force experienced by particle n and V_i is the total volume of particle i.

The DM and MDM models require very little computational power and run time but fail at low particle distances. FVM, while providing good results for the whole range of separation distances, requires high computational time. Therefore, an empirical model of force between two particles is developed.

Emperical Model: The force between identical two spherical particles in a uniform magnetic field is dependent on the following parameters: magnitude H_0 and direction of applied magnetic field $\hat{\mathbf{H}}_0$, particle radius a, separation distance c, and particle volume magnetic susceptibility χ. Looking at the formulation of the solver, we can conclude that the force between the particles is

$$F \propto a^2 H_0^2 \tag{8}$$

Two special cases are considered separately for applied magnetic field direction: magnetic field parallel and perpendicular to the line joining the particles. Finally, based on the data set prepared through the spherical harmonics solver, empirical formulas for separation distance (expressed in particle radius) and magnetic susceptibility are developed. The data is fitted for the equation

$$F = F_{DM} + \mu_0 H_0^2 a^2 \sum_{i=1}^{3} \frac{p_i}{c^{4+i}} \tag{9}$$

where F_{DM} is the force between particles when only a fixed dipole model is considered and parameters p_i are dependent on susceptibility. As relation to H_0 and a are known we generate data with both assumed to be unity. Then, the separation data for each susceptibility is fitted on equation 8 to get parameters. Based on the curve of each parameter p_i vs susceptibility χ, either an exponential or a polynomial function is fitted for each parameter. The empirical model is also tested for extrapolated data with R^2 value within 0.95-1.00 for all cases.

Meteorite Data: Kohout et al. [12] measured bulk magnetic susceptibility and bulk densities for over 700 meteorite samples. The volume magnetic susceptibility range for stony-iron and iron meteorites is 5-150. We generated 5-100 susceptibility data to fit the function and checked for extrapolation for 0.05-5 and 100-150.

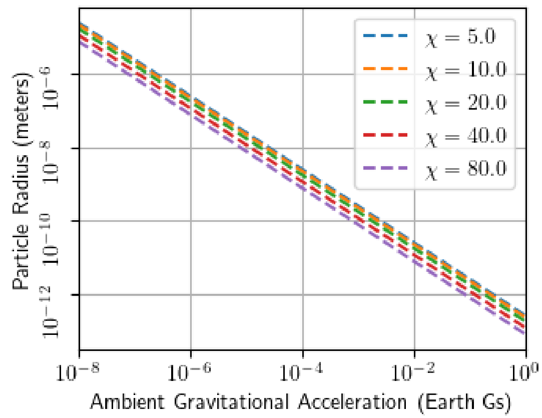

Figure 1: Radii of surface particles for ambient weight equal to magnetic force as a function of ambient G. (assumed magnetic field & particle density are 500 nT & 4 g/cm^3 respectively)

Conclusions: The bond number (ratio of cohesive force by particle weight) for the magnetic cohesive force between two identical spherical particles with radius r in contact is given by

$$B_m = \frac{\mu_0 H_0^2}{\rho g a} f(\chi) \tag{10}$$

where ρ is the particle density, g is ambient gravity and $f(\chi)$ is the function of magnetic susceptibility (different for parallel and perpendicular magnetic field direction cases). This provides a means to characterize magnetic force extending the work in [5] (Fig. 1). The empirical model facilitates quick calculation of force between particles at close distances. For larger separation distance MDM is used. This combination enables us to perform discrete element method (DEM) simulations with multiple particles of various sizes in low-gravity, low-pressure environment of asteroids to investigate the role of magnetic cohesion in regolith on such asteroids.

Acknowledgments: We would like to thank Dr Thomas Leps for his preliminary work on this project.

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