

Libration-Induced Orbit Period Variations Following the DART Impact. A. J. Meyer¹, D. J. Scheeres¹, I. G. Gkolias², M. Gaitanas², H. F. Agrusa³, K. Tsiganis², P. Pravec⁴, L. A. M. Benner⁵, F. Ferrari⁶, and P. Michel⁷, ¹University of Colorado Boulder, ²Aristotle University of Thessaloniki, ³University of Maryland, ⁴Astronomical Institute of the Czech Academy of Sciences, ⁵Jet Propulsion Laboratory, ⁶Universität Bern, ⁷Université Côte d'Azur.

Introduction: The Double Asteroid Redirection Test (DART) mission will be the first test of a kinetic impactor as a means of planetary defense [1]. In Fall 2022, DART will collide with Dimorphos, the secondary in the Didymos binary asteroid system. The impact will cause a momentum transfer from the spacecraft to the binary asteroid, changing the orbit period of Dimorphos and forcing it to librate in its orbit. Owing to the coupled dynamics in binary asteroid systems, the orbit and libration state of Dimorphos are intertwined [2]. Thus, as the secondary librates, it also experiences fluctuations in its orbit period. These variations in the orbit period are dependent on the magnitude of the impact perturbation, as well as the system's state at impact and the shape of the secondary. In general, any binary asteroid system whose secondary is librating will have a non-constant orbit period on account of the secondary's fluctuating spin rate. The orbit period variations are typically driven by two modes: a long-period and short-period, each with significant amplitudes on the order of tens of seconds to several minutes.

Owing to the proximity and irregular shapes of the bodies in a binary asteroid, these systems are driven by coupled attitude and orbit dynamics in what is known as the full two-body problem [3]. This problem has been studied in depth [4], and we particularly rely on the work in [2], who study the libration in these systems. More accurate numerical models have also been developed, and we rely on the General Use Binary Asteroid Simulator (GUBAS) for accurate simulations of binary asteroid dynamics [5].

Dynamics Model: In this work we take advantage of two dynamics models for binary asteroids. The first is a simple two-dimensional planar model, where the primary is modeled as an oblate spheroid and the secondary as a triaxial ellipsoid. This model is used to establish a link between the secondary's libration rate and the mutual orbit period. Assuming the combined angular momentum of the secondary's spin and the mutual orbit is constant, this takes the form of a basic conservation of angular momentum, and is written as

$$\dot{\theta} = \frac{H - I_z \dot{\lambda}}{I_z + \nu r^2}$$

where H is the constant angular momentum, I_z is the secondary's polar moment of inertia, ν is the mass fraction of the binary system, and r is the separation distance. This relationship demonstrates how the orbit rate and secondary spin rate are related, and the relevant

system parameters that affect this relationship. At an equilibrium value of angular momentum, the secondary spin rate is zero (tidally locked) and the orbit rate is constant. However, any deviation from this equilibrium value will have a varying orbit rate, and as a result a non-constant orbit period. It is worth noting that the issue of defining the orbit period in a binary asteroid system is non-trivial. Owing to the coupled nature of the full two-body problem, the dynamics are non-Keplerian. Thus, we define the orbit period as the time required for the secondary to complete one full rotation relative to an inertial plane.

Assuming Didymos is currently in an equilibrium state. The DART impact will serve as a perturbation to this equilibrium, where the change in angular momentum is defined as

$$\Delta H = \beta \nu M_{DART} V_{DART}$$

where M_{DART} is the mass of the impactor and V_{DART} is its velocity relative to Dimorphos. β is called the momentum enhancement factor and is an important quantity in the DART mission. Indeed, one of the primary objectives of the mission is to determine this factor. Simply put, β is the ratio of the total momentum transferred to the momentum transferred by a perfectly inelastic collision. This captures the effect that ejecta from the surface of the target will have on the overall momentum transfer. Thus, the minimum value of β is 1 corresponding to an inelastic collision with no ejecta and the sole transfer of momentum from the spacecraft to the asteroid. The true value of β will be larger than 1 and dependent on the properties of the body.

The second model used in this analysis is the more accurate GUBAS code. This uses a fourth degree and order gravity field along with polyhedral models for the asteroids to propagate the full equations of motion of the system. Thus, while the first model is a highly simplified representation of binary asteroids, GUBAS will serve as a check on this model and a means of obtaining more accurate results, including three-dimensional effects.

Results: To simulate a DART impact, we start the binary system in an equilibrium configuration with the exception of its perturbed angular momentum value for a given β factor. While propagating the equations of motion, we calculate the exact times the secondary has crossed the inertial y - z plane and difference these times to calculate each independent orbit period. The choice of this plane is arbitrary. Figure 1 shows the orbit period

of Didymos over time for the case $\beta = 3$. This clearly shows how the orbit period varies over time.

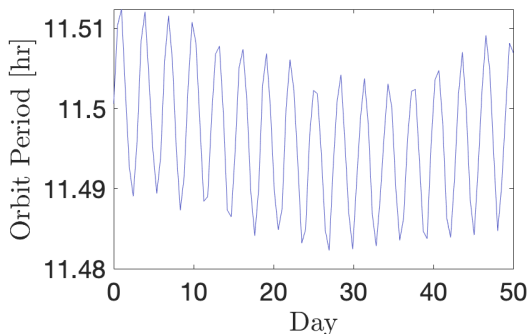


Figure 1: The orbit period over time for the post-impact Didymos system with a value $\beta = 3$

We define the orbit period variation as the elementary range of the time history shown in Figure 1; that is, the difference between the maximum and minimum orbit period over the time domain. In this way, we can describe the orbit period variations as a scalar and expand our analysis. We vary the value of β between 1 and 5 and calculate the corresponding orbit period variation and libration amplitude. These results are shown in Figure 2, along with data points calculated using the high fidelity GUBAS results for validation.

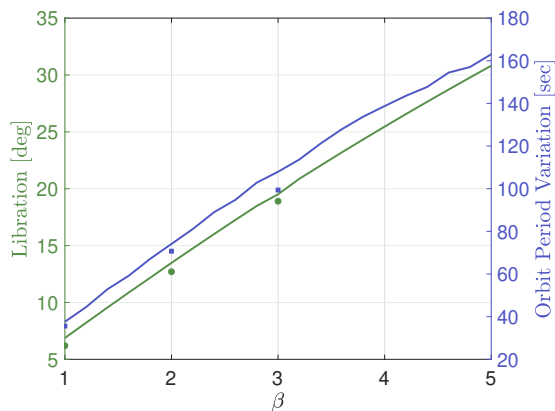


Figure 2: The libration amplitude and orbit period variation for varying values of β . The data points are corresponding GUBAS results.

In Figure 2 we see a clear linear relationship between β and both the libration amplitude and orbit period variation. This again illustrates how closely related the secondary spin and orbit period variations are.

While this establishes the DART impact will result in a non-constant orbit period, there are several additional factors that warrant further investigation. Namely, the shape of Dimorphos is poorly defined, and so this analysis is repeated for a wide range of secondary shapes. The current configuration of Didymos is also currently unknown and may not be in equilibrium. Thus,

several initial conditions should also be tested to examine how a non-equilibrium configuration affects the post-impact dynamics.

Lastly, note in Figure 1 that the orbit period variations appear to be driven by two modes: a short-period and a long-period. These modes strongly depend on the shape of the secondary and the value of β . As an illustration, Figure 3 shows the orbit period of three shapes of Dimorphos, defined by the ellipsoidal axis ratio a/b . This shows how drastically the behavior can change with the secondary's shape and highlights the importance of obtaining an accurate shape model.

These modes are driven by the apsidal precession of the eccentricity vector, measured by the Keplerian longitude of periapsis. On average, this precesses with a frequency equal to the long-period mode, with short-period oscillations within that precession.

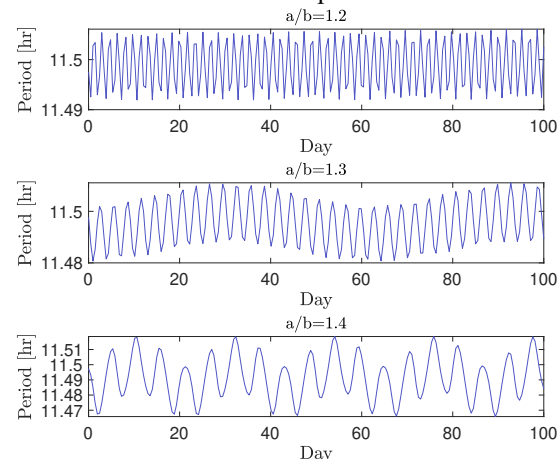


Figure 3: The orbit period variations for three different shapes of Dimorphos.

These modes are particularly important as they carry direct implications for post-impact observations. For example, it is prudent that observations span for the duration of the long-period mode. This way, observations have the best chance of detecting variations in the orbit period, as the short-period mode may be too short to detect.

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