ADVANCED HYPERBOLA FITTING, REVERSE-TIME MIGRATION AND CLUSTERING: AN ADVANCED SET OF TOOLS FOR MAPPING THE INTERNAL STRUCTURE OF LUNAR SOIL USING GROUND PENETRATING RADAR. I. Giannakis[1], J. Martin-Torres[1,2], M-P., Zorzano[3], C. Warren[4] and A. Giannopoulos[5]. [1] University of Aberdeen, Meston Building, Kings College, Aberdeen, UK, AB24 3FX, email: iraklis.giannakis@abdn.ac.uk, javier.martin-torres@abdn.ac.uk, [2] Instituto Andaluz de Ciencias de la Tierra (CSIC-UGR), Armilla, Granada, Spain, [3]Centro de Astrobiología (INTA-CSIC), Torrejón de Ardoz, 28850 Madrid, Spain, email: zorzanomm@cab.inta-csic.es, [4]Department of Mechanical and Construction Engineering, Northumbria University, Newcastle, UK, NE1 8ST, email: craig.warren@northumbria.ac.uk [5] School of Engineering, The University of Edinburgh, Edinburgh, EH9 3FG, UK, email: a.giannopoulos@ed.ac.uk.

Introduction: Ground-penetrating radar (GPR) is a mature geophysical methodology with a wide range of applications, from non-destructive testing and hydrogeology to glaciology and landmine detection [1]. Its minimum operational requirements combined with its ability to operate in dry low-conductivity media, made GPR the most mainstream geophysical technique in planetary science [2]. Although orbiter sounders were employed both for lunar and Martian exploration since the early 70s, it would take several decades for the first conventional in-situ GPR to be deployed in Chang'E-3 mission [3]. In the following years, GPR has been used in Chang'E-4 [4], E-5 [5], Tianwen-1 [6] ,Perseverance [7] and it is planned to be used in future missions such as Chang'E-7 (expected to be launched in 2024) and ExoMars (expected to be launched in September 2022). In the current paper, we describe a coherent interpretation scheme that utilises an advanced hyperbola fitting scheme to infer the velocity structure of the lunar regolith at the Chang'E-4 landing site. Subsequently, the estimated velocity structure is used in reverse-time migration (RTM) coupled with finite-differences time-domain (FDTD) method, capable of focusing the signal subject to any arbitrary layered media. Lastly, the reconstructed migrated image is clustered in order to accurately map the subsurface targets (rocks and boulders) in the investigated medium.

Methodology: The arrival time from a spherical target with radius R buried in a homogenous medium at point $\mathbf{B}$ is given by $t = \frac{2}{c_0}\sqrt{\epsilon}\,(||\mathbf{A}-\mathbf{B}|| - R)$, where $\mathbf{A}$ is the vector with the coordinates of the GPR unit, $c_0 = 3\times10^8$ m/s is the velocity of light, and ϵ is the relative permittivity of the medium (assumed homogenous). Conventional hyperbola fitting tries to find the optimum ϵ that minimises the error between the measured arrival times $\mathbf{T} \in \mathbb{R}^n$ and the predicted ones $\mathbf{t} \in \mathbb{R}^n$, where n is the number of measurements.

For a layered medium with small permittivity variations (ignoring diffraction phenomena) the two way arrival time equals with $t = \frac{2||\mathbf{A}-\mathbf{B}||-R}{c_0 d}\int_0^d \sqrt{\epsilon(z)}dz$ where d is the depth of the target. The latter can be re-written as $t = \frac{2}{c_0}\sqrt{\epsilon_b}\,(||\mathbf{A}-\mathbf{B}|| - R)$ where $\sqrt{\epsilon_b}$ is the bulk/average square root of the relative permittivity from the surface down to the depth of the target d.

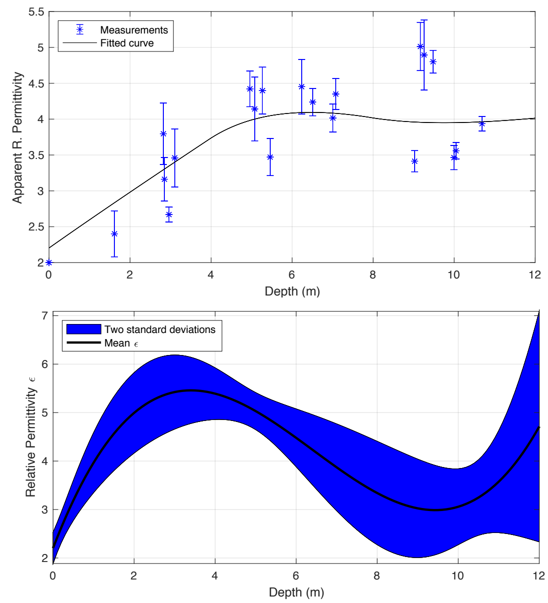

Figure 1: Top: The measured bulk permittivities using the stochastic hyperbola fitting; and the fitted permittivity of the inverted model. Bottom: The estimated permittivity profile with its uncertainty bounds.

Therefore, by applying a conventional hyperbola fitting in N targets, we can estimate the average square root of the relative permittivity for different depths $\sqrt{\epsilon_b(d_i)}$ $i \in [1, 2, 3...N]$.

Conventional hyperbola fitting suffers from non-uniqueness in the presence of noise. In particular, multiple sets of $\{\epsilon_b(d), d, R\}$ can give rise to similar arrival times, making it difficult to simultaneously estimate both the permittivity of the host medium and the radius of the target [8]. To mitigate that, typical hyperbola fitting assumes a point target ($R = 0$), a rough assumption that can greatly influence the estimated permittivity. In the current paper, first we discretise the hyperbola manually into w points $\mathbf{T} \in \mathbb{R}^w$. Subsequently, particle swarm optimisation (PSO) is used to minimize $\underset{\epsilon_b,d,R}{\operatorname{argmin}}||\mathbf{T}-\mathbf{t}||$. The mean $\mu = E[\mathbf{T}-\mathbf{t}]$ and the standard deviation $\sigma = \sqrt{E[(\mathbf{T}-\mu])^2}$ is assumed to be the level of noise

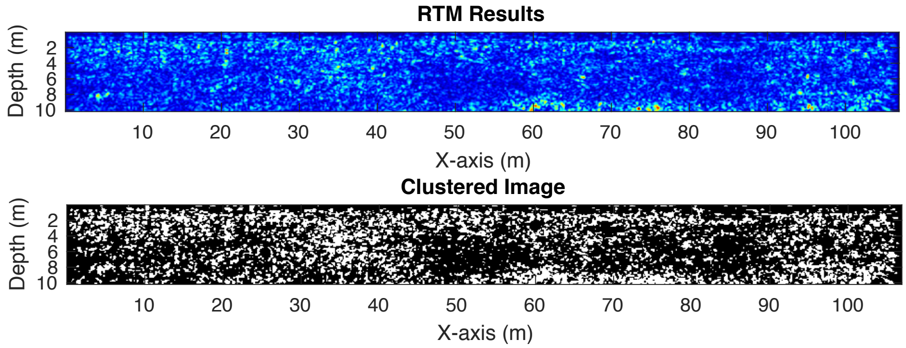

Figure 2: Top: The post-migrated image using RTM for the permittivity profile shown in Fig. 1. Bottom: Segmenting the image using the threshold methodology suggested in [10].

in our observations. Based on that, a new measurement vector $\mathbf{T}$ is derived which equals with the fitted hyperbola plus a Gaussian noise $\mathbf{T} = \mathbf{t} + \mathcal{N}(\mu,\, \sigma^2)$. The minimisation $\underset{\epsilon(d),R}{\operatorname{argmin}} |||\mathbf{T} - \mathbf{t}||$ is executed again with the new observations $\mathbf{T}$ and a new set of $\{\epsilon_b(d), d, R\}$ is derived. The procedure is repeated for sufficient amount of times in order to calculate the uncertainty of $\epsilon_b(d)$ and d. Via this procedure instead of a single permittivity and depth, the stochastic hyperbola fitting estimates the normal distribution of $\epsilon_b(d) \sim \mathcal{N}(\mu_{\epsilon_b},\, \sigma^2_{\epsilon_b})$ and $d \sim \mathcal{N}(\mu_d,\, \sigma^2_d)$.

The next step is to infer the 1D permittivity profile of the medium based on $\mathcal{N}(\mu_{\epsilon_b},\, \sigma^2_{\epsilon_b})$ and $\mathcal{N}(\mu_d,\, \sigma^2_d)$. Discretising the permittivity in m equidistant depths $\{\epsilon(z_1), \epsilon(z_2)...\epsilon(z_m)\}$, and using spline functions to interpolate between different depths, results in $\epsilon(z) = P_i(z)$ where $z \in [z_i, z_{i+1}]\ \forall i \in [1, 2, 3...m-1]$, and $P_i(z)$ is a third order polynomial. The proposed scheme tries to find the best values of $\epsilon(z_i)$ that minimise $\underset{\epsilon(z_i)\forall i\in[1,m]}{\operatorname{argmin}} \sum_{j=1}^{m} \left(\sqrt{\epsilon_b(d_j)} - \frac{1}{d_j}\int_0^{d_j}\sqrt{\epsilon(z)dz}\right)^2$. The latter is executed numerous times using PSO, each time the ϵ_b and d values are chosen randomly subject to their statistical properties (i.e. $\mathcal{N}(\mu_{\epsilon_b},\, \sigma^2_{\epsilon_b})$, $\mathcal{N}(\mu_d,\, \sigma^2_d)$) as estimated via the stochastic hyperbola fitting described in the previous paragraph. The resulting $\epsilon(z_i)\forall i \in [1, 2, 3..m]$ for each PSO execution are then used to estimate the uncertainty of the reconstructed permittivity distribution. In particular, we derive a Gaussian distribution $\mathcal{N}(\mu_{\epsilon(z_i)},\, \sigma^2_{\epsilon(z_i)})\forall i \in [1, 2, 3..m]$ that describes the probability of a permittivity profile to be true.

The average permittivity profile μ_{ϵ_z} is then used in RTM coupled with FDTD, in order to focus the processed Bscan, increase the overall signal to clutter ratio and map subsurface targets. RTM propagates the waves back in time at $s = 0$, where the waves collapse in their reflection sources. The filtered B-Scan is denoted as $B(\mathbf{q}, \mathbf{s})$, for $\mathbf{s} \in [0, s_{max}]$ and $\mathbf{q}_k = ||\langle x_k,\ y_k,\ z_k\rangle||$, where $\{x_k, y_k, z_k\} \in \mathbb{R}$ are the coordinates of the kth measurement. The first step in RTM is that $B(\mathbf{q}, \mathbf{s})$ is reversed in time $B(\mathbf{q}, s_{max} - \mathbf{s})$. The reversed traces are subsequently used as impressed current sources $J_u(\mathbf{q}, \mathbf{t}) = B(\mathbf{q}, s_{max} - \mathbf{s})$ where $u \in \{x, y, z\}$ is the polarisation of the receiver. To address the two-way travel time, the velocity of the medium is set to half the actual velocity. A TM-FDTD with second order of accuracy in both space and time is used in the current paper. Similar to [9], the post-migrated image is further processed by taking its absolute value and smoothing it by applying a Gaussian 2D filter.

Finally, the resulting post-migrated image is clustered using a threshold selection method from gray-level histograms as proposed in [9]. In the current paper, two different clusters are considered to represent the host medium and the buried rocks and boulders.

Results: The proposed scheme is applied to the first 100 meters of GPR data collected at the Von Kármán crater by the Yutu-2 rover [11]. Figure 1 shows the estimated bulk permittivity using the stochastic hyperbola fitting; and the estimated permittivity profile. In Figure 2, the permittivity profile shown in Figure 1 is used as input to the RTM using TM-FDTD. The post-migrated image is subsequently segmented using the methodology suggested in [9].

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