TIDES AND THE SPIN STATES OF THE ICY OCEAN WORLD EUROPA. E. R. Burnett[1] and P. O. Hayne[2,3], [1]Aerospace Engineering, University of Colorado Boulder (email: ethan.burnett@colorado.edu), [2]Astrophysical and Planetary Sciences, University of Colorado Boulder, [3]Laboratory for Atmospheric and Space Physics, University of Colorado Boulder

Introduction: Geologic evidence suggests that the icy surface of Europa could undergo either episodic or slow continuous reorientation with respect to the tidally locked state, and this has significant implications for the study of the global ice shell and the subsurface ocean [1]. The continuous reorientation solution is of particular interest, and has been the subject of several notable works [see e.g. 2,3].

Classical analyses of tidal locking [4,5] use a 1D single-axis model of a planet as a rigid body, and may be inaccurate for understanding the motion of a decoupled ice shell. The icy ocean world Europa is well-known to be differentiated into an ice shell and core, partially dynamically decoupled by a subsurface ocean. A 2D model of independent single-axis ice shell and core movements thus presents a more appropriate first-order model, while retaining a useful degree of simplicity. This work discusses such an approach using a model previously derived for studying ice shell librations [6]. The new approach provides insight to questions about the spin state of Europa, and critically, the degree to which its ice shell and core can move independently. In addition, the potential for distributed geologic intrusions and mass concentrations in the ice shell to produce or alter dynamically significant mass asymmetries is also discussed.

Background: It has been known since the works of Goldreich and Peale [4,5] that a planet in an eccentric orbit can settle into a spin rate that is slightly faster than the synchronous angular velocity. In the final spin state, the orbit-averaged tidal torque should equal zero, and this happens for a planetary spin rate value between the periapsis angular velocity ω_p and the mean motion angular velocity n. However, if the planet has sufficient permanent mass asymmetry, then gravity-gradient torques on this asymmetry can dominate the tidal torque and force synchronous rotation. The condition for initially super-synchronous rotation to settle into a libration-only synchronous final spin state is given by the following relationship between orbit eccentricity e and planetary principal moments of inertia $C > B > A$:

$$\left(\frac{3}{2}\left(\frac{B-A}{C}\right)\right)^{\frac{1}{2}} > \frac{9.5\pi e^2}{2\sqrt{2}} \tag{1}$$

In the case of Europa and other icy ocean worlds, the ice shell and core can move independently to at least some degree, being dynamically decoupled by a global ocean. If the ocean is sufficiently deep, even full independent relative rotations are presumably possible. For this problem, constraints on the final spin states should depend on the properties of both the ice shell and core, so the simple conclusions of Goldreich and Peale [4,5] need to be modified.

In Goldreich and Peale's analysis, the dynamics of the planar angle from the periapsis direction to the long axis of the planet θ are those of a nonlinear pendulum, with external harmonic forcing due to the effects of orbit eccentricity. The study of the dynamics of the averaged angle $\eta = \theta - nt$ (where n is the mean motion) is a useful simplification for this problem, given by the autonomous differential equation below in the absence of tidal torques:

$$C\ddot{\eta} + \frac{3}{2}(B-A)n^2 \sin 2\eta = 0 \tag{2}$$

Note $|\eta| < \frac{\pi}{2}$ if the body is tidally locked. The above equation admits an energy integral E [see e.g. 4] which, along with careful use of averaging, crucially enables simple analysis of the long-term evolution of the rotational state.

In the case of Europa, there are two angles to track, θ_s and θ_i, for the ice shell and interior (core) respectively. This is depicted in Figure 1 below.

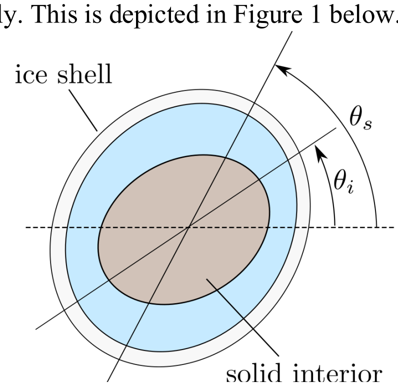

Fig 1. An icy ocean world differentiated into an ice shell and solid interior, separated by a global ocean (not to scale)

As noted in [6], the dynamics of the averaged angles $\eta_s = \theta_s - nt$ and $\eta_i = \theta_i - nt$ are given by the following coupled equations:

$$\begin{aligned} C_s\ddot{\eta}_s + \frac{3}{2}(B_s - A_s)n^2 \sin 2\eta_s + K_G \sin\big(2(\eta_s - \eta_i)\big) = 0 \\ C_i\ddot{\eta}_i + \frac{3}{2}(B_i - A_i)n^2 \sin 2\eta_i - K_G \sin\big(2(\eta_s - \eta_i)\big) = 0 \end{aligned} \tag{3}$$

where the coupling constant K_G is due to the gravity-gradient torque between shell and core. This analysis neglects viscous effects of the ocean layer [see e.g. 7] but these are assumed to be comparatively small [6]. Critically, Eq. (3) is equally valid for libration, as studied previously in [6], and for full rotation (i.e. unbounded η_s and η_i), which was not explored.

Analysis: Just as Eq. (2) provides insights into the characteristic behaviors possible in the 1D spin-orbit coupling problem, Eq. (3) is useful for the differentiated problem. First, it can be shown that there is still an energy integral for the more complicated averaged 2D model, Eq. (3), and it is given below:

$$E = \frac{C_i}{2}\dot{\eta}_i^2 + \frac{C_s}{2}\dot{\eta}_s^2 - \frac{3}{4}(B_i - A_i)n^2\cos 2\eta_i - \frac{3}{4}(B_s - A_s)n^2\cos 2\eta_s - \frac{1}{2}K_G\cos\big(2(\eta_s - \eta_i)\big) \tag{4}$$

This expression turns out to be very useful, as will be discussed further below.

It is worth noting that $\frac{C_i}{C_s} \sim 200$ for Europa, which limits the types of characteristic behaviors that are possible. The much more massive core behaves more regularly than the ice shell. For example, at certain energy levels, energy exchange between the interior and exterior can conceivably induce irregular rotation in the ice shell, while the interior simply librates with minimal perturbation. This is illustrated below in results from an example numerical simulation of the unforced 2D dynamics given in Eq. (3).

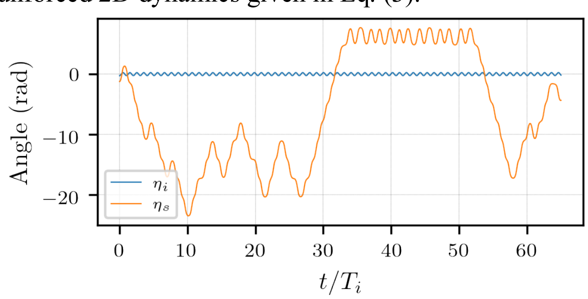

Fig 2. Complex ice shell spin behavior, for an example with strong gravitational coupling and ${C_i}/{C_s} \sim 200$. The steady librations of the much more massive interior are $|\eta_s| < 16°$.

Despite the complexity exhibited by this system, the total energy of the averaged system can provide a useful perspective. To study how the total system energy relates to the permissible spin states, the concept of the *zero-velocity curve* is borrowed from celestial mechanics [8]. Namely, setting angular velocities to zero and examining the curves of constant values of E, for a given energy level, the range of reachable angles $\{\eta_i, \eta_s\}$ is externally bounded by the curve at that fixed energy value. This is depicted in Figure 3. As energy is increased from its lowest levels in the plot, full rotation of the ice shell becomes possible well before it is possible for the interior. In particular, it is possible to calculate from Eq. (4) the critical values of E below which full rotation of the ice shell and interior are impossible. These are given as $E_{c,1}$ and $E_{c,2}$ in Figure 3, where $E_{c,2} \gg E_{c,1}$.

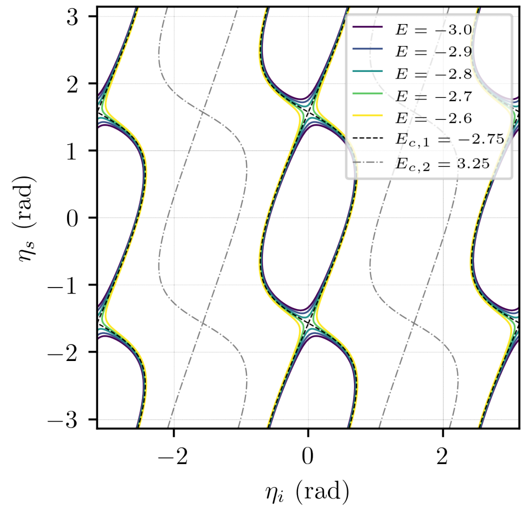

Fig 3. Example zero-velocity curves, showing cutoffs for full rotation of ice shell ($E > E_{c,1}$) and interior ($E > E_{c,2}$).

This analysis suggests that if the moon starts in a high energy fully super-synchronous rotation state, energy loss due to the transient tidal torques will tidally lock the interior before the ice shell. In an extension of Goldreich's classical analysis [4], averaging of the tidally forced problem over the period of motions of the interior could provide additional insights. Numerical exploration of this simple system is also a viable approach, further facilitated by insights from the 2D averaged system [6].

The complex unsolved problem of the rotation and orientation history of Europa's ice shell is one of coupled orbital and tidal dynamics, and also general 3D rotations. It will also require consideration of surface features to look for signatures of rotation, when possible [see e.g. 9,10]. However, simplified models can be quite useful, particularly when they provide reasonable constraints, and the above analysis is thus more relevant for Europa and similar worlds than the classical works of Goldreich and Peale.

References: [1] Greenberg, R. et al. (2002) *CMDA, 83,* 35–47. [2] Hoppa, G. V. et al. (1999) *Science, 285,* 1899–1902. [3] Ojakangas, G. W. and Stevenson, D. J. (1989) *Icarus 81,* 242–270. [4] Goldreich, P. (1966) *TAJ, 71, no. 1.* [5] Goldreich, P. and Peale, S. (1966) *TAJ*, *71, no. 6.* [6] Van Hoolst, T. et al. (2008) *Icarus, 195,* 386–399. [7] Hay, H. C. F. C. et al. (2020) *GRL, 47,* issue 15. [8] Koon, W. S. et al. (2011) *Springer*. [9] Tajeddine, R. et al. (2017) *Icarus, 295,* 46–60. [10] Burnett, E. R. and Hayne, P. O. (2021) *Icarus, 364,* article no. 114438.