THERMAL MODELS OF YORP AND YARKOVSKY EFFECTS: TYPICAL EVOLUTION AND YORP EQUILIBRIA. O. Golubov[1], V. Unukovych[1], D. J. Scheeres[2], A. V. Kopatko[1], A. Strelchenko[1], [1]V. N. Karazin Kharkiv National University (4 Svobody Sq., Kharkiv, 61022, Ukraine, oleksiy.golubov@karazin.ua), [2]Ann and H.J Smead Aerospace Engineering Sciences, University of Colorado Boulder (3775 Discover Dr, 80303, USA).

Introduction: YORP and Yarkovsky effects, which are the governing forces of dynamical evolution of small asteroids [1], have been analytically expressed in [2] as integrals of universal p-functions over the surface of an asteroid. The p-functions are determined by the thermal model of the material on the asteroid surface, and expressed as some averages of the surface temperature.

Here, we apply this formalism to different asteroid shapes, study their typical YORP evolution and evaluate their Yarkovsky effect. Importantly, we find an equilibrium rotation state, in which the YORP acceleration is zero, and which originates due to complicated dependence of the obliquity component of YORP on the rotation rate.

Methods: We create two programs for thermal modeling of the asteroid subsurface layer. Both programs solve the one-dimensional heat conductivity equation in a semi-space with the non-linear boundary condition. The latter includes the impinging solar radiation, the heat conduction to the depth of the asteroid, and the thermal emission from the surface. One program implements the finite difference method, whereas the other one makes use of the Fourier decomposition of the temperature as a function of depth and time, and choses the coefficients of the decomposition to satisfy the non-linear boundary condition at the highest possible accuracy.

The used formalism of the p-functions intrinsically assumes a convex shape of the asteroid. To evaluate the importance of this assumption, we create a ray-tracing program, which can thoroughly calculate the Yarkovsky and YORP effects for asteroids of arbitrary shapes, although at present only for a vanishingly small thermal conductivity.

For our study, we use 5716 shape models from DAMIT obtained by lightcurve inversion technique [3,4], 29 shape models obtained by radar imaging [5], and 4 shape models of asteroids Bennu, Eros, Itokawa, and Ryugu obtained by the space missions [6].

Results: We fit an analytic expression to the result of numeric simulations of the p-functions, and substitute the result into the representation [2] of the YORP effect as a surface integral. As an outcome, we get an analytic expression of the YORP effect that includes two coefficients, $C_{\cos}$ and $C_{\sin}$, which have to be determined numerically for each given asteroid shape. This formalism generalizes the approach developed in [7] that described the case of zero thermal inertia. In the latter case, the YORP evolution

of an asteroid could be described using only one numerically determined coefficient, which is proportional to $C_{\sin}$. On the other hand, $C_{\cos}$ is a new entity, which arises only in the case of non-zero thermal inertia.

The two coefficients, $C_{\cos}$ and $C_{\sin}$, appear to be uncorrelated. $C_{\sin}$ is due to the propeller-like asymmetry, whereas $C_{\cos}$ is nonzero even for symmetric asteroids, and can be attributed to their flattening and, to a lesser extent, to the roughness of their surface. As most asteroids are relatively symmetric, $C_{\cos}$ is typically an order of magnitude larger than $C_{\sin}$, with the result that asteroid's obliquity tends to 0 or 180 degrees. Noteworthy, this behavior holds only for asteroids with thermal parameters close to unity, whereas in other cases the large value of $C_{\cos}$ is suppressed by the vanishingly small value of the analytic function by which it is multiplied. Thus we estimate that the widely assumed fast YORP alignment of asteroid equatorial planes with their orbital planes holds only for the minority of objects.

If $C_{\cos}$ and $C_{\sin}$ have the same sign, it is possible for the obliquity component of the YORP effect to change sign as a function of the rotation rate. The corresponding evolutionary diagram is sketched in Figure 1. The lines where either the obliquity component of YORP τ_ε or the axial component τ_z are zeros are marked with dashed lines. At very small rotation rates (the lower-left corner of the diagram)
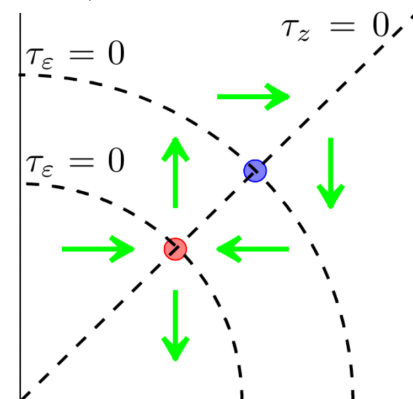

Figure 1. Sketch of the asteroid evolution in a two-dimensional plane, where the polar radius represents the rotation rate and the polar angle represents the obliquity. The dashed curves mark the lines where the axial or obliquity components of the torque are zeros. The green arrows denote the directions of the dynamical evolution. The red and blue circles are the unstable and stable equilibria correspondingly.

and very large rotation rates (large polar radii) the term C_{sin} dominates, producing the behavior similar to the one described in [7] for the zero thermal inertia case.

Two intersection points in the diagram, where both of the YORP components are zeros, correspond to the YORP equilibria. The lower one, marked by the red circle, is unstable saddle point, as it can be seen from the green arrows. The upper one, marked by the blue circle, is a focal point. Unfortunately, our simplified analytic equations do not allow us to make conclusion about stability of this focal point: they are separable, resulting into closed trajectories. A slight perturbation of the equations can turn the equilibrium in either asymptotically stable or unstable one. Our full numeric simulations without any analytic approximations show that at least in some cases this equilibrium is indeed stable.

Such stable equilibria have already been discussed by [8], although in a simplified model, where the thermal modeling of the asteroid was substituted by the assumption of a fixed thermal lag between the absorption and re-emission of energy by the asteroid surface. These equilibria can have important evolutionary consequences and serve as attractors, to which asteroids can sink as a result of their YORP evolution. This adds to the equilibria arising from other physical mechanisms, which have been previously discussed in the literature [7,9,10].

Lastly, we use the formalism of the *p*-functions to compute the Yarkovsky effect of individual asteroids and to parameterize the Yarkovsky force as a function of the asteroid shape, obliquity, and thermal properties.

Conclusions: We construct an approximate analytic description of the YORP and Yarkovsky effects by boiling down an inclusive physical model to a few free parameters describing the asteroid shape. We use a sample of asteroid shapes to study the statistics of these coefficients. This gives us a powerful tool to simulate evolution of asteroid populations in a simple but robust way. Such tool would help us to understand asteroid delivery to the near-Earth orbits, asteroid grinding by the YORP cycles, and evolution of asteroid families.

References: [1] Vokrouhlicky D. et al. (2015) *Asteroids 4*. [2] Golubov O. et al. (2016) *MNRAS 458*, 3977. [3] Durech J. et al. (2010) *A&A 513*, A46. [4] DAMIT, https://astro.troja.mff.cuni.cz/projects/damit/. [5] Asteroid Radar Research. Asteroid Shape Models https://echo.jpl.nasa.gov/asteroids/shapes/shapes.html. [6] PDS Small Bodies Node. Shape Models of Asteroids, Comets, and Satellites, https://sbn.psi.edu/pds/shape-models/. [7] Golubov O., and Scheeres D. J. (2019) *AJ 157*, 105. [8] Scheeres D. J. and Mirrahimi S. (2008). *Celestial Mechanics and Dynamical Astronomy 101*, 69. [9] Golubov O., and Scheeres D. J. (2016) *ApJL 833*, L23. [10] Golubov O. et al. (2018) *ApJL 857*, L5.