FINITE ELEMENT MODELING APPROACH TO CHARACTERIZE TEMPERATURE VARIATIONS OF IRREGULARLY SHAPED BODIES. R. Nakano[1] and M. Hirabayashi[1], [1]Department of Aerospace Engineering, Auburn University, 211 Davis Hall, Auburn, AL 36849-5338 (rzn0040@auburn.edu).

Summary: We are developing a new thermophysical model based on Finite Element Modeling (hereafter, TPFEM) to describe the temperature variations of airless bodies' structure, given illumination conditions and physical properties. We applied the TPFEM to asteroid (162173) Ryugu and obtained the surface temperature consistent with earlier studies. The TPFEM will be a powerful tool to investigate the Yarkovsky and Yarkovsky-O'Keefe-Radzievskii-Paddack (YORP) effects.

Introduction: Thermophysical models have been developed to quantify the thermal environments of planetary bodies [e.g. 1, 2, 3]. For small bodies, quantifying the thermal conditions are particularly significant because such conditions control the dynamic behaviors of these bodies, leading to orbital drifts (Yarkovsky effect) and attitude perturbations (YORP effect) [4]. Therefore, proper thermophysical modeling with proper boundary conditions such as shadowing, scattering of sunlight, self-heating, and surface roughness can constrain the dynamic evolution of such objects.

Thermophysical models in earlier works usually accepted global shape models or local topographic models composed of triangular facets [e.g., 2, 5]. While implementation might be different for each model, only 1D heat diffusion equation has often been considered under the assumption that the lateral heat conduction is negligible. Also, investigations of the Yarkovsky and YORP effects often used the "Rubincam's approximation" for simplification [6], where the thermal inertia is assumed to be zero, and thus, the surface temperature is simply found by the energy balance at the boundary (thus, no need to solve heat diffusion equation). However, this approximation is not suitable for the Yarkovsky effect and the YORP induced pole shift investigations [7].

Finite Element Modeling (FEM)-based Thermophysical Model: We are currently developing a FEM-based thermophysical model (TPFEM). Unlike earlier works which typically solve the 1D heat diffusion equation by using a finite difference scheme or a Newton-Raphson scheme [e.g. 2, 5], this model solves the 3D heat diffusion equation by using a FEM approach. The TPFEM accepts a global shape model expressed in tetrahedral elements and takes into account shadowing, scattering of sunlight, and self-heating, similar to other thermophysical models. The surface roughness is not yet considered in the current version.

FEM formulation of 3D heat diffusion equation – The FEM formulations for the 3D heat diffusion equation are derived using Galerkin's method. The integral form of the 3D heat diffusion equation is given by:

$$\iiint_V \rho c \frac{\partial T}{\partial t} dV = -\iint_S \boldsymbol{n}\cdot\boldsymbol{q} dS - \iint_S \boldsymbol{Q}\cdot\boldsymbol{n} dS, \quad (1)$$

where ρ is the density, c is the specific heat capacity, T is the temperature, V is the volume, $\boldsymbol{q}$ is the heat flux at the boundary, S is the area, $\boldsymbol{Q}$ is the heat energy at the boundary, and $\boldsymbol{n}$ is the outward positive normal vector on the surface. Using the Galerkin's method, which allows us to determine temperature of an element, T_j $j = 1, \dots, m$, Eq. (1) can be expressed as:

$$\left(\sum_{j=1}^{m} \iiint_{V_j} \left(\rho c \frac{\partial T_j}{\partial t} + \boldsymbol{\nabla}\cdot\boldsymbol{q}\right) dV_j + \sum_{j=1}^{m} \iint_{S_j} \boldsymbol{Q}\cdot\boldsymbol{n} dS_j\right)\phi_j = R, \quad (2)$$

where ϕ_j is the element shape functions, and R is the residual. For the sake of simplicity, we omit the detailed derivations here and only show the final expression which describes the temperature of the element:

$$\sum_{j=1}^{m} \left(\boldsymbol{A}_j \dot{\boldsymbol{T}}_j + \boldsymbol{B}_j \boldsymbol{T}_j + \boldsymbol{C}_j\right) = \boldsymbol{R}, \quad (3)$$

where

$$\boldsymbol{A}_j = \iiint_{V_j} \rho_j c_j \boldsymbol{N}_j \boldsymbol{N}_j^T \, dV_j, \qquad \boldsymbol{B}_j = \iiint_{V_j} k_j \boldsymbol{P}_j \boldsymbol{P}_j^T dV_j,$$

$$\boldsymbol{C}_j = \iint_{S_j} Q_j \cdot n_j \boldsymbol{N}_j^T dS_j,$$

where $\boldsymbol{N}$ and $\boldsymbol{P}$ are the shape function matrices. Constructing the global matrices from the element matrices and setting the residual vector $\boldsymbol{R}$ to be zero, we obtain the global system of ordinary differential equations which can be easily solved by many numerical methods.

Tetrahedral mesh model – In FEM, the resolution of mesh model is one of the important parameters that control the quality of the final result. Generally, the higher the resolution, the better the result. However, the computational burden can easily become large if we simply consider high resolution tetrahedral mesh models of the sizes of NEAs, leading to undesirably long computational times. To mitigate this issue, we make use of the fact that for principle axis rotators, the thermal skin depth L is substantially smaller than the radius of the body. Also, L is smaller than the sizes of topographic features [5]. The thermal skin depth, which is given by $L = \sqrt{4\pi P_{rot} k/(\rho c)}$ where P_{rot} is the spin period and k is the heat conductivity, is the depth at which the amplitude of temperature variation have decreased by a factor of $\sim 10^{-3}$ [8]. Given this fact, we construct "hollow" tetrahedral mesh model – only the layers of tetrahedral meshes (shell) exist for the surface and subsurface, and the inside is left as cavity. The most inner nodes are fixed at a constant temperature in the TPFEM. To mitigate the influence of the constant temperature constraint, we set the thickness of the shell to be larger than at least $2L$. This mesh generation technique significantly reduces the computational burden, while maintaining the resolution of the surface and subsurface meshes.

Application to Asteroid (162173) Ryugu: We used the TPFEM to describe the surface temperature of asteroid (162173) Ryugu. The orbital and physical parameters at the epoch of 1 July 2018 was used [9]. The "hollow" tetrahedral mesh model was generated based on the shape model by Watanabe et al. [9]. The thermal inertia was set to ~200 J m^{-2} K^{-1} $s^{-1/2}$ (hereafter, tiu). The thermal skin depth L was found to be 0.15 m. We run a simulation with an initially uniform temperature distribution of 280 K at all the defined nodes. We confirmed that the temperature evolution immediately reached a steady-state condition in which the temperature periodically changes over the defined asteroid rotation. In the present case, such a condition appeared after 1 revolution.

Figure 1 shows the surface temperature distribution of Ryugu simulated by the TPFEM. Figure 2 shows the time history of nodal temperature for 1 rotation (1 Ryugu's day). While there is a slight difference in the maximum/minimum temperature due to differences in parameter settings (i.e. values of density, specific heat capacity, rotation period, etc.) as well as the coordinates of the selected node, the temperature amplitude of ~100 K for ~200 tiu with a mean temperature of ~280 K is consistent with that obtained by other thermophysical models [i.e. 10, 11]. However, this result is not consistent with the measured brightness temperature profile (i.e. flat diurnal temperature) of Ryugu, which may be caused by a strong surface roughness effect [11].

Discussion and Summary: The TPFEM has shown several advantages over other thermophysical models.

Firstly, the TPFEM employs the 3D heat diffusion equation. Because it does not impose the assumption of no lateral heat conduction, the TPFEM can describe more realistic thermal interactions between neighboring facets, which may be important in the investigation of subsurface (or boulders) temperature evolutions as well as Yarkovsky and YORP effects.

Secondary, while earlier thermophysical models often enforced to determine the surface temperature by using a Newton-Raphson iterative technique, this treatment may violate the heat equation in Eq (1) (though, a justification was made by [8]). The TPFEM does not need such processes because this can be avoided in the FEM approach and the computation becomes straight forward.

Lastly, the TPFEM can be relatively easily combined with other FEM simulation packages. For example, by incorporating the FEM structural simulation package [12], it is possible to investigate thermal-structural interactions of surface materials of airless bodies. Similarly, it is also possible to incorporate the FEM approach Full Two-Body Problem simulation package (F2BPFEM; [13]) in order to investigate the Binary YORP effect [14].

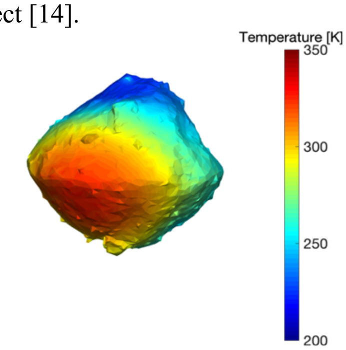

Figure 1. Surface temperature distribution of asteroid (162173) Ryugu simulated by the TPFEM.

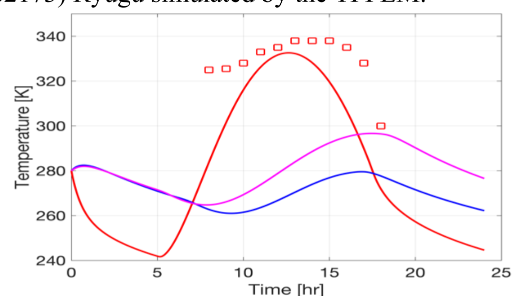

Figure 2. Time history of the surface temperature. The red, blue, and magenta lines correspond to the latitude of the nodes: 16°S, 40°N, and 30°S, respectively. The longitude is almost consistent for all the nodes (~0°). The brightness temperature measured by Hayabusa2 is plotted in red squares (extracted from [11]).

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