

TOWARDS MODELING CRATER COLLAPSE AND RELAXATION WITH A STOKES-FLOW CODE . D. G. Korycansky, CODEP, Department of Earth and Planetary Sciences, University of California, Santa Cruz CA 95064 .

Introduction

The Galilean satellites with icy surfaces (Ganymede, Callisto, Europa) are host to a variety of large impact features that are, if not unique to these bodies, rarely encountered on planetary and satellite surfaces in the Solar System. These features include impact basins with central pits, domes, and so-called “penepalimpsests” and “palimpsests” in the terminology of Schenk *et al.* 2004. Our project seeks to establish the effects of several factors in explaining the origin and evolution of these features. In particular we aim to establish the roles played by: 1) the presence or absence of liquid water (at depth below the surface, or generated during the impact) vs warm ice (again, either pre-existing or impact-generated), 2) the lithospheric temperature gradient, 3) surface gravity (as compared to smaller gravity on mid-sized satellites, where the features of interest are not found, and finally 4) the role of the characteristics of the impactor: specifically, the impactor’s size, velocity, composition, and the angle of the impact.

This abstract describes work carried out by the author (Korycansky) who is the project PI. In particular we describe work towards modeling crater relaxation using a multi-fluid Stokes-flow program that we are developing. The goal of the work is to complement our other efforts in crater modeling and to understand how various target rheologies and structures influence crater relaxation. In this we follow the precedent of Ivanov and Kostuchenko (1997), who used a free-surface fluid-dynamics code combined with models for post-impact acoustic fluidization to test ideas about impact crater morphology.

The program follows the methodology described by Schmeling and Marquardt (1991) and Weinberg and Schmeling (1992). We use a stream-function formulation in the high-viscosity limit of the incompressible Navier-Stokes equation in two-dimensional cartesian coordinates (x, z) . The time-dependent and inertial terms of the equations are neglected, so that buoyancy terms (resulting from density differences in a gravitational field) are balanced by viscous terms, which determines the flow field. Time evolution then occurs by material movement via the continuity equation. Substitution of the stream function ψ into the viscous terms of the Navier-Stokes equation and taking the curl results in a elliptic-type fourth-order partial differential equation for the stream function:

$$4 \frac{\partial^2}{\partial x \partial z} \left(\eta \frac{\partial^2 \psi}{\partial x \partial z} \right) + \left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2} \right) \left[\eta \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial x^2} \right) \right] = g_z \frac{\partial \rho}{\partial x},$$

where ρ is the density of the fluid, g_z the vertical gravity, and η is the dynamic viscosity. Note that the equation may be non-linear if η is a function of the stream function or its derivatives, as may be the case for a power-law fluid where the viscosity is a function of the strain-tensor product.

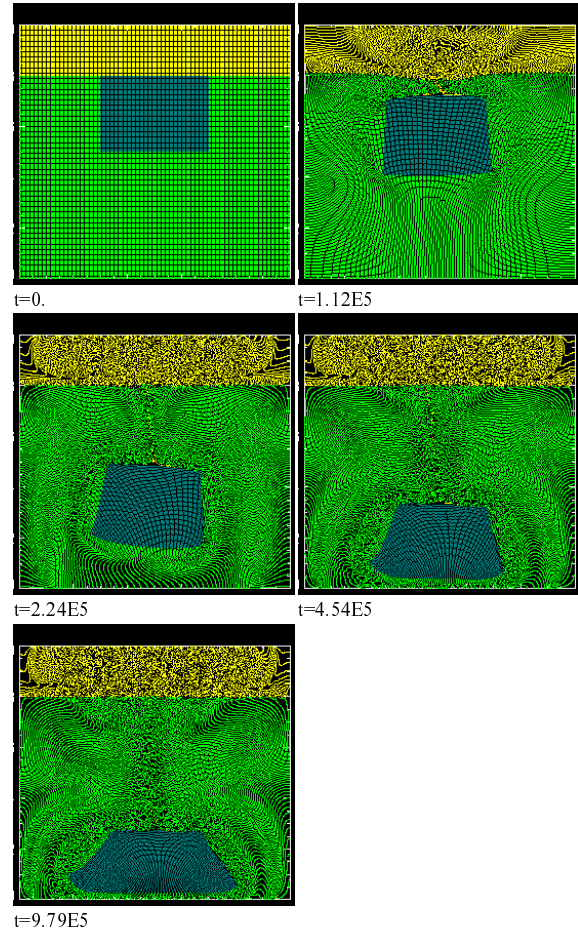


Figure 1: Sample calculation for Stokes stream function program. The calculation models the sinking of a highly viscous block into less dense, less viscous fluid, below a low-density “atmosphere”. Marker particles (yellow, green, blue) show the placement of three fluids. Three fluids are involved: 1) low-density, low-viscosity fluid (yellow markers) $\rho_1 = 1$, $\eta_1 = 1$ (kinematic viscosity $\nu_1 = \eta_1/\rho_1 = 1$), 2) high-density, high-viscosity fluid (green markers) $\rho_2 = 90$, $\eta_2 = 9.0 \times 10^3$ (kinematic viscosity $\nu_2 = \eta_2/\rho_2 = 100$), and 3) high-density, very-high-viscosity fluid (blue markers) $\rho_3 = 10^2$, $\eta_3 = 1.0 \times 10^6$ (kinematic viscosity $\nu_3 = \eta_3/\rho_3 = 10^4$). The background grid (not shown) is 50×50 in size and the total number of markers is 4×10^4 . The vertical gravity $g = 1$, so the highest-density fluid slowly sinks and eventually spreads at the bottom impermeable free-slip boundary.

The continuity equation is solved via Lagrangian marker particles whose positions are updated by movement according to the velocity field. Labeling particles with different types

allows the use of multiple fluids with differing properties such as density or viscosity. In this formulation, the entire numerical grid is filled, so there is no free surface as such, but large density contrasts (e.g. 1000 to 1) can serve the same purpose. Non-Newtonian fluids can be included via non-constant effective viscosity that is a function of the strain tensor product or other fluid quantities.

Numerical solution of the stream function equation is carried out by first discretizing on a grid, resulting in a linear system $\mathbf{Ax}=\mathbf{b}$, where \mathbf{A} is the coefficient matrix, \mathbf{b} is a vector of the right-hand side terms of the stream function equation and \mathbf{x} is the solution vector of the stream function on the grid. The coefficient matrix \mathbf{A} is sparse and symmetric, and can be written via Cholesky decomposition as the product \mathbf{LL}^T , where \mathbf{L} is a (still relatively sparse) banded lower triangular matrix and \mathbf{L}^T is its transpose. Efficient solution via Cholesky decomposition allows for feasible iteration schemes when the viscosity η is a function of the strain product, but it is apparently often the case that the most straightforward Picard-type iteration schemes converge slowly or result in non-converging oscillations (Izmail-Zadeh and Tackley 2010). For the moment we will present some qualitative results where cases in which the viscosity is a function of fluid-type only. Figure 1 shows an example of the settling of a slightly higher-density viscous block ($\rho = 10^2$, dynamic viscosity $\eta = 10^6$) into the background of slightly lower density, less viscous fluid. ($\rho = 90$, $\eta = 9 \times 10^3$). Above both fluids is a low-density, low-viscosity fluid ($\rho = 1$, $\eta = 1$) that approximately models a free-surface interface (so-called “sticky air”, Izmail-Zadeh and Tackley 2010). The background grid (not shown) is 50×50 in size and the total number of markers is 4×10^4 .

Present status

At the time of writing, we have developed a working multi-fluid program. As with previous papers mentioned above,

the program is coded for two-dimensional flow in cartesian coordinates. Development for two-dimensional axisymmetric cylindrical coordinates is ongoing; the matrix coefficients for the axisymmetric stream function equation are complicated but incorporating them into the cylindrical coordinate version of the code is eminently feasible. More challenging may be the development of a robust iteration scheme for the implementation of non-Newtonian viscosity that will allow consistent calculation of rheologies that can model physical processes like acoustic fluidization or other complex behavior that occurs during crater relaxation on medium to long timescales (Melosh 1989). Preliminary tests with non-iterative scheme (where the viscosity is determined by the flow field previous timestep) show that a simple scheme may not be adequate for these calculations.

Acknowledgments

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