The long-term maintenance of liquid-water habitats by self-tuned ocean tidal resonance
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Liquid water oceans in the universe may be far more stable, long-lived, and abundant than previously thought. This conjecture is not simply an extrapolation from surprising recent discoveries in our Solar System. Rather, it comes from considerations of the internal fluid dynamical response of a generic ocean to tidal forces, and feedbacks from this response that stabilize ocean parameters against secular trends. Relatively basic dynamical arguments are combined to show that attempting to freeze or stratify an ocean pushes it toward a resonant state, with an increase in dissipative heating and mixing that counters these trends and stabilizes the ocean over long periods of time. The aim of this presentation is to provide a short and simple description and demonstration of this important dynamical effect for the broad community currently developing a path forward in the investigation of ocean worlds.

To illustrate the self-tuning effect, we integrate (using an explicit Runge-Kutta method) over non-dimensional time t' a simple ice-growth model of the form $\frac{d\tilde{h}_I}{dt'} = k_1 \tilde{P}_{radio} + k_2 \tilde{c}_e^2 \tilde{P}_{tidal} - k_3 \tilde{P}_{cool}$, where $\tilde{P}_{radio} = e^{-t'}$ is an exponentially decaying (e.g. radiogenic or solid-tidal) heat source, $\tilde{P}_{cool} = 1/\tilde{h}_I$ represents a simple (e.g. conductive) cooling, and $\tilde{P}_{tidal}$ is the non-dimensional ocean tidal power calculated using the TROPF software package (see Abstract # 2883 by this author). The dimensionalization of time as well as the choice of coefficients $k_{i=1,2,3}$ depend on the specific case. For the generic illustration here, we arbitrarily choose $k_1 = 1$, $k_3 = k_1/\tilde{h}_I(t' = 0)$, $k_2 = 2 \times 10^2 k_3$. (The basic points we wish to illustrate will be robust provided the dimensionalized tidal power is strong enough to counterbalance the dimensionalized cooling and that the time span is long enough for the $\tilde{P}_{radio}$ source to decay below these levels.) One may view the terms $\tilde{P}$ as representing average power density per volume, with the factors k_1, k_2 incorporating the constant radial integration factors. The tidal term, however, has the additional factor $\tilde{c}_e^2$ (as discussed next) to account for the variation with time of the ocean depth over which this source is integrated.

The tidal power $\tilde{P}_{tidal}$ depends on the tidal force, but also the parameters controlling the ocean's tidal response. We consider first the simplest case for forcing which is the situation of the ocean spinning rapidly relative to the orbit of a tide raising body in a circular, equatorial orbit. In this case, the force is represented by simply one propagating spherical harmonic term (a degree-two spherical harmonic propagating across the ocean in the retrograde sense with twice the ocean's rotation frequency. We assume in this case that the ocean's response is governed by the Laplace Tidal Equations, with dissipation proportional to the kinetic energy density, and varies with two parameters $\tilde{T}$, $\tilde{c}_e^2$, where $\tilde{T}$ is the ratio of dissipation and tidal-period time scales, and $\tilde{c}_e$ is the ratio of ocean wave speed to twice the equatorial rotation speed. TROPF is then used to calculate 261522 solutions sampling the tidal response over a range of $\tilde{T}$, $\tilde{c}_e^2$. For each pair, the time/globe averaged power (heating rate) is plotted to produce Figure 1 A. The first result from this figure is that the solutions include both very low and very high power tidal scenarios that depend sensitively on the input parameters $\tilde{T}$, $\tilde{c}_e^2$.

Under the assumption of barotropic tides, $\tilde{c}_e^2$ is proportional to the ocean's fractional thickness $\tilde{h}$ (the total water+ice thickness is $\tilde{h} + \tilde{h}_I = 1$) and the dependence of $\tilde{c}_e^2$ on the evolving ice thickness is then $\tilde{c}_e^2 = (1 - \tilde{h}_I)\tilde{c}_e^2(t' = 0)$. We also assume here that as the ice thickens, this has stronger damping on the ocean tides, decreasing $\tilde{T}$. For the example here, we assume the simple linear dependence $\tilde{T} = (1 - \tilde{h}_I)\tilde{T}(t' = 0)$ which represents extreme overdamping as the ocean approaches a fully frozen state. The second result (shown in Fig 1 A,B,C) is that instead of continuing to freeze as $\tilde{P}_{radio}$ decays, the ocean tides increase in power to ultimately stall freezing near a resonant configuration, with the ocean parameters and heating becoming remarkably stable following this adjustment.

Of course, the final fraction of ice as well as the evolution path depends on the relative values of these arbitrarily assigned parameters. Increasing the relative amplitude of the tidal-power can lead to a stable ocean with less ice, and vice versa. If damping is assumed to increase more quickly with ice

thickness than in the model assumed here, we can expect the path arm leading from the asterix in Fig. 1A to rotate toward the 9:00 position. Whereas with damping independent of ice it would rotate toward the 6:00 position. In any case, the secular freezing causes a migration that leads to increased tidal heating and the precise point of equilibrium is rather a detail in the concept we wish to convey here.

Conclusion: When considering the preponderance and stability of liquid water oceans in the Solar System and beyond, and for constructing an efficient research path forward for ocean worlds, self-tuned tidal resonance should be included and closely examined. Basic dynamics as well as the demonstration provided here suggest that oceans in systems with even weak tidal forces are remarkably hard to freeze. This has a large impact on our starting assumptions for ocean worlds on icy satellites in the Solar System, as well on exoplanets and interstellar nomads.
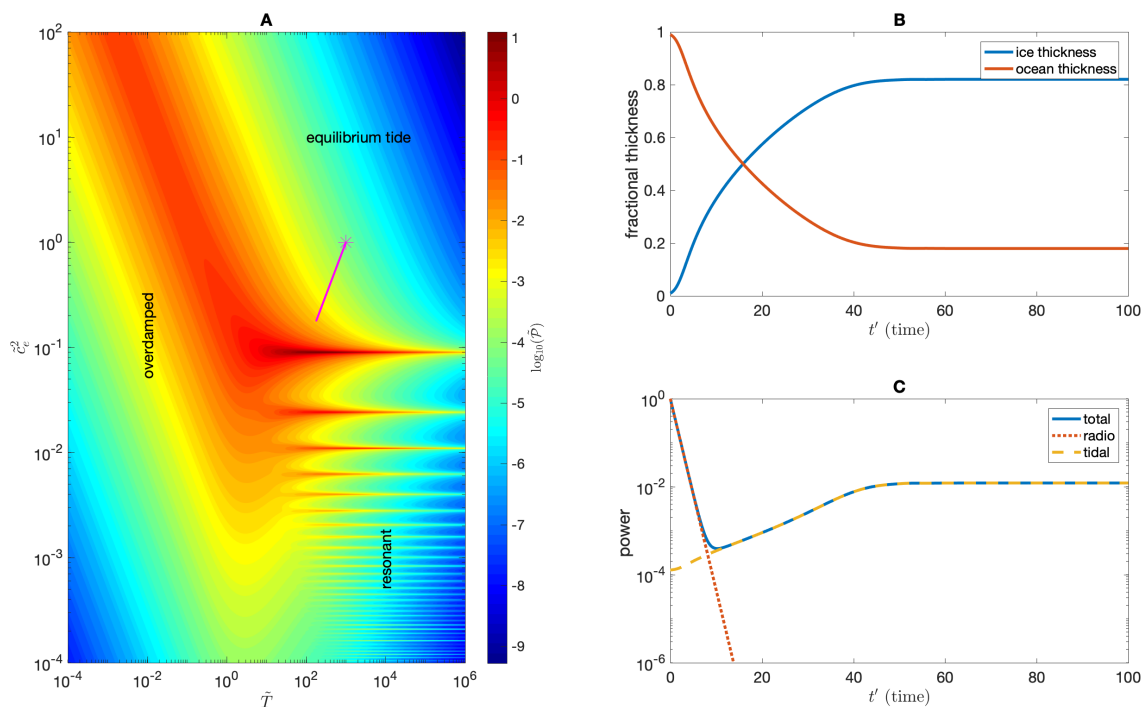

Figure 1: A: Nondimensional tidal heating rate (i.e. ocean depth-integrated power $\tilde{c}_e^2 \tilde{P}$ on a $\log_{10}$ scale) as a function of the ocean's nondimensional squared wave speed ($\tilde{c}_e^2$) and dissipation timescale ($\tilde{T}$). Here, barotropic tides are assumed, in which case $\tilde{c}_e^2$ is proportional to the ocean's fractional thickness $\tilde{h}$ (the total water+ice thickness is $\tilde{h} + \tilde{h}_I = 1$). Superimposed is an example evolution path of an ocean with initial parameter coordinates (asterix) for which there is small tidal heat contribution. Initially, the ocean has negligible ice ($\tilde{h}_I(t' = 0) = 10^{-2}$) and is maintained by a source (labeled 'radio' in C) that decays exponentially with time and leads to a tendency for secular cooling, ice growth and therefore a migration toward lower $\tilde{h}$ and $\tilde{c}_e^2$. There is also the tendency to migrate toward lower $\tilde{T}$ as it is assumed that damping increases with ice thickness. The migration is stalled at a point where increased tidal heat counters the cooling. This self-tuned effect is remarkable in stabilizing the ocean state and power/heat delivered (as seen in B and C), in contrast to radiogenic or solid tidal heat sources which typically show a continual decay with time.