

Dynamics of two-phase flows inside geyser conduits on Enceladus. A. Carballido¹, ¹Center for Astrophysics, Space Physics and Engineering Research, Baylor University, One Bear Place #97283, Waco, TX, 76798-7283, USA.

Introduction: Plume material on Enceladus may offer a glimpse of sub-surface processes. For example, *Cassini* plume observations can be used to elucidate the dimensions of vertical conduits in the Enceladus ice shell, as well as mechanisms responsible for plume variability, by developing conduit flow models of vapor-ice wall interaction [1]. Likewise, the velocities with which ice particles leave surface vents are probably the result of complex dynamics within Enceladus' ice fractures. Collisions between particles and conduit walls may slow down large grains compared to smaller grains and vapor [2].

Despite early attempts at understanding the inner workings of Enceladus' plumbing, many questions about the flow dynamics in the satellite's conduits remain unanswered. What flow regimes does water vapor experience as it ascends through conduits? How do waves propagate in the vapor flow? Are shocks produced at any point?

This contribution seeks to address questions such as these by employing a two-phase formalism developed in the context of terrestrial volcanic eruptions [3]. The approach, which in turn is based on a general model of the dynamics of two-phase mixtures [4], is used to analyze the propagation of acoustic and porosity waves inside ice shell conduits on Enceladus, as well as the vertical speed of ice particles there.

Overview of the model: The material inside an Enceladus conduit is assumed to consist of a mixture of continuous water vapor and dispersed water ice particles. The mixture originates from a sub-surface vapor reservoir at the water triple point. The model allows for phase separation and interaction [3].

The governing equations of the 1-D model are (see [3] for a detailed derivation):

Mass conservation

$$\frac{\partial(\rho_v \phi)}{\partial t} + \frac{\partial(\rho_v \phi w_v)}{\partial z} = 0, \quad (1a)$$

$$\frac{\partial(1-\phi)}{\partial t} + \frac{\partial[(1-\phi)w_v]}{\partial z} = 0; \quad (1b)$$

Momentum conservation

$$\rho_v \phi \left(\frac{\partial w_v}{\partial t} + w_v \frac{\partial w_v}{\partial z} \right) = -\phi c_s^2 \frac{\partial \rho_v}{\partial z} - \rho_v \phi g + C \Delta w + \frac{1}{2} \tilde{\rho} \left(\frac{\partial \Delta w}{\partial t} + \tilde{w} \frac{\partial \Delta w}{\partial z} \right); \quad (2a)$$

$$\rho_i (1-\phi) \left(\frac{\partial w_i}{\partial t} + w_i \frac{\partial w_i}{\partial z} \right) = -(1-\phi) c_s^2 \frac{\partial \rho_v}{\partial z} - \rho_i (1-\phi) g - C \Delta w - \frac{1}{2} \tilde{\rho} \left(\frac{\partial \Delta w}{\partial t} + \tilde{w} \frac{\partial \Delta w}{\partial z} \right). \quad (2b)$$

The symbols are listed in Table 1. Ice is treated as incompressible. The system is isothermal. A linear analysis of Eqs. (1)-(2) allows one to ascertain the effect that the motion of an eruptive column has on the propagation of acoustic waves. A non-linear treatment, requiring a numerical solution, yields the behavior of $\phi(z)$ and $w_i(z)$. In both approaches, a steady state is assumed.

Table 1: symbols used in Eqs. (1) and (2)

Symbol	Meaning
z	Vertical coordinate
ρ_v, ρ_i	Vapor density, ice density
ϕ	Vapor volume fraction (porosity)
w_v, w_i	Vapor/ice vertical speed
c_s	Vapor sound speed
g	Gravitational acceleration
C	Drag coefficient [3]
Δw	Vapor-ice relative vertical speed
$\tilde{\rho}, \tilde{w}$	Effective interface density/speed

Results: Acoustic and porosity waves. The linear analysis of the system (1)-(2) gives a dispersion relation from which the propagation speed of pressure waves can be calculated. Fig. 1 shows this wave speed as a function of wavelength, for different values of the reservoir vapor volume fraction ϕ_0 . At short

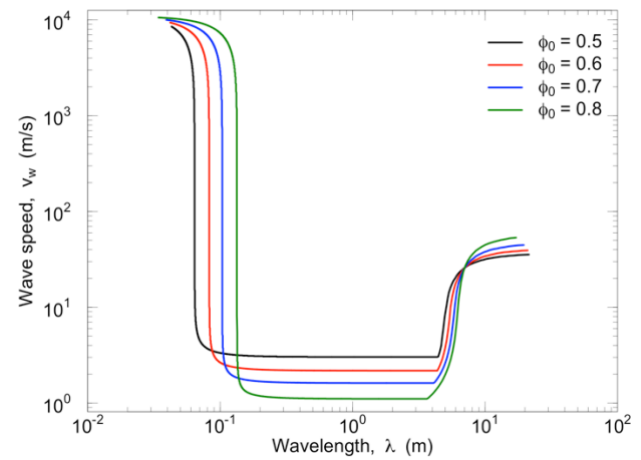


Figure 1: Speed of propagation of acoustic waves as a function of wavelength, within a two-phase mixture of water vapor and ice flowing vertically under Enceladus' conditions.

wavelengths, acoustic modes travel close to, or above, the water vapor sound speed (taken as $c_s \sim 400$ m/s; e.g., [2]). At long wavelengths, the speed of propagation is subsonic, and it may correspond to porosity waves, i.e., pulses of buoyant vapor volume fraction, in analogy to volcanic conduits [3]. The wave speed is attenuated at intermediate wavelengths.

Vertical profiles of porosity and ice speed. Fig. 2 shows the behavior of porosity ϕ along the height of a conduit, for two values of the initial eruption speed \mathcal{W}_0 (parameterized by $\bar{\alpha} \equiv c_s/\mathcal{W}_0$: 293 m/s ($\bar{\alpha} = 1.4$, Fig. 2a) and 684 m/s ($\bar{\alpha} = 0.6$, Fig. 2b). Each curve corresponds to a different value of the initial porosity ϕ_0 at the reservoir (i.e., at $z = 0$). Notice that, in all cases, the vapor volume fraction remains unchanged up to a certain height, and increases monotonically at larger distances from the reservoir. In the $\bar{\alpha} = 0.6$ case, ϕ_0 values above 0.7 produce an unstable solution.

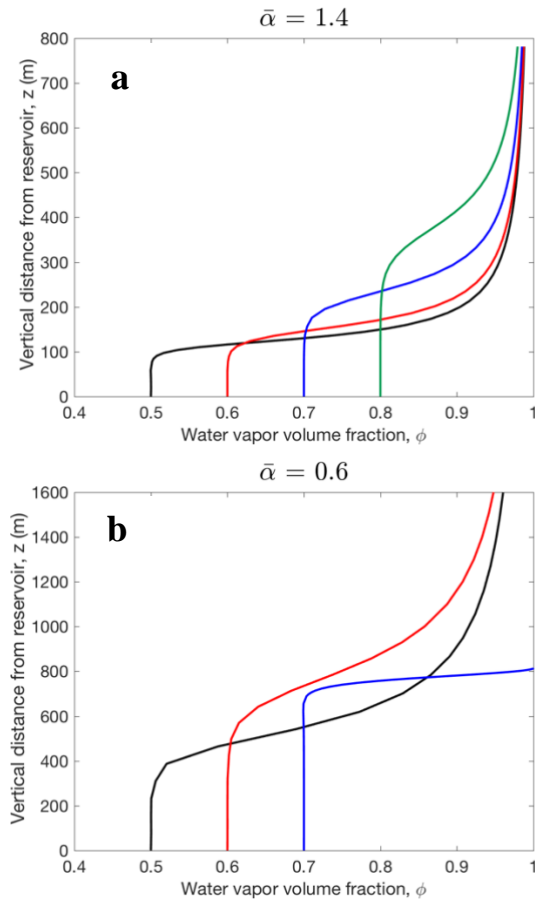


Figure 2: Vertical profiles of vapor volume fraction, for two values of initial eruption speed \mathcal{W}_0 (parameterized by $\bar{\alpha} \equiv c_s/\mathcal{W}_0$): 293 m/s ($\bar{\alpha} = 1.4$, Fig. 2a) and 684 m/s ($\bar{\alpha} = 0.6$, Fig. 2b). Each curve corresponds to a different reservoir porosity at $z = 0$ (note different vertical scales in both panels).

The speed of ice grains, whose radii are constant at $1 \mu\text{m}$, is shown in Fig. 3 for the same two values of $\bar{\alpha}$ as

in Fig. 2, and for different reservoir porosities ϕ_0 . In the case of a slow eruption ($\bar{\alpha} = 1.4$, Fig. 3a), the largest grain speeds are attained at lower heights above the reservoir than for a fast eruption ($\bar{\alpha} = 0.6$, Fig. 3b).

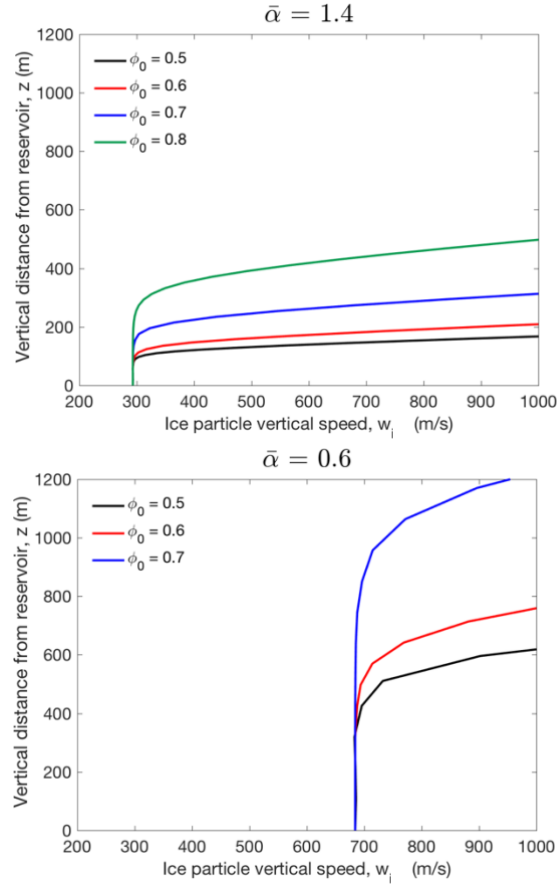


Figure 3: Vertical profiles of ice grain speed, for the same two values of the initial eruption speed parameter $\bar{\alpha}$ as in Fig. 2. Each curve corresponds to a different reservoir porosity at $z = 0$.

Discussion: The very high particle speeds computed above is a consequence of the 1D assumption. The irregularity of ice conduits [2] would introduce a 3D character to the particle motion, whereby ice grains would lose momentum as they collide with nonuniform walls.

The vertical behavior of porosity may be a result of two-phase effects like compaction, compression and interphase drag, as found in (terrestrial) volcanic conduits [3].

Future work will investigate the role of ice particle size distributions.

References: [1] Nakajima, M., Ingersoll, A. P. (2016), *Icarus*, 272, 309. [2] Schmidt, J. et al. (2008) [3] Bercovici, D., Michaut, C. (2010), *Geophys. J. Int.*, 182, 843.