**Dynamics of Mars Lake-overflow Valley Incision.** Samuel J. Holo and Edwin S. Kite

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**Introduction:** Valley and valley-network fed paleo-lakes are common on Mars [1,2] and are important for constraining late-Noachian climate and surface processes [3,4] In particular [3] inferred from the presence of outflow valleys for valley-network-fed paleo-lakes [1] that mean annual runoff production in the late-Noachian was large enough for large crater-lakes to overspill and breach their rim. Thus, wet periods in the late-Noachian must have not been so arid or short that the paleo-lakes never filled (e.g. [5]). While it is clear that filling lakes to the brim is necessary, it remains unclear if it is sufficient. In principle, overflow valley formation to the observed depth might require sustained runoff from the inlet valley that greatly exceeds the lake-filling volume (requiring a correspondingly longer wet climate). Alternatively, the depth of overflow valleys may result mainly from rapid incision following a single catastrophic lake-draining event. Thus, developing an understanding of the mechanics of outflow valley erosion is necessary for determining to what extent paleo-lake outflow valleys record climatic forcing.

Relatively little modeling work has been done on Martian paleo-lake overspill erosion but previous studies (e.g. [6]) have used the methodology of [7] in which flow through the breach is assumed to be critical (Fr = 1). While this approximation works well for constructed-dam failure and weir flow, it does not account for flow resistance through the breach, which is significant for landslide dams with larger along-valley length scales [7.8]. Where flow resistance is important, explicit coupling of flow resistance, sediment transport, and lake draining in models produces more accurate results [8]. These models are common for flood hazard assessment but are limited to modeling erosion of dams that are short compared to their host bedrock valleys [8]. Thus, these models do not necessarily provide insight into what controls the erosion of long overflow valleys on Mars.

Geologists have long been interested in modeling the formation of large bedrock valleys by past megafloods on Earth (e.g. [9]). However, models of bedrock valley formation by megaflood have typically prescribed flow depth in the valley is to be either brim-full or just above the threshold for erosion (due to morphodynamic adjustment) without considering dynamics of the lake level in the upstream source (e.g. [9]). Here, we introduce a new mathematical model for the formation of lake overspill bedrock valleys that dynamically couples flow resistance, lake draining, and sediment transport. We will demonstrate with our model that flow depth in the valley can exhibit behavior more complex than the near-threshold or brim-full scenarios.
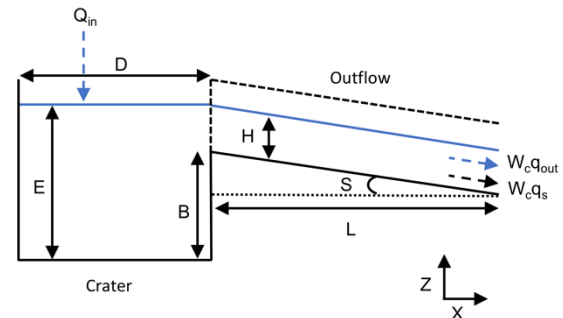


**Figure 1.** Simple schematic showing the model cross-sectional geometry. Initial breach and outflow valley topography are represented by the dashed lines. In this scenario, conservation of mass describes the time evolution of $H$ as a function of source fluid discharge into the lake ($Q_{in}$), water/sediment flux per unit channel-width through of the distal end of the valley ($q_{out}$ and $q_s$), channel width, $W_c$, valley width, $W_v$, valley length, $L$, and lake diameter, $D$

**Mathematical Model:** Our goal is to write down the simplest possible model that conserves mass, incorporates realistic flow resistance and sediment transport equations, and couples lake draining to breach erosion (in particular, erosion of the breach allows more water within the lake to be released). To achieve this, we consider a model for flow depth in the channel that is made 0-dimensional by (a) ignoring along-valley changes in flow properties and channel/valley geometry and (b) approximating the geometry of the lake as a cylinder (Figure 1). This yields an ordinary differential equation that is non-dimensionalized in terms of the transport stage (ratio of shear stress to critical shear stress, $T = \frac{\tau_*}{\tau_{*c}}$):

$$\frac{dT}{dt_*} = K_2 - K_1 T^{5/3} + (\max(T, 1) - 1)^{3/2}$$

where $t_*$ is dimensionless time,

$$K_1 = \frac{R^{7/6} \tau_{*c}^{1/6}}{S^{7/6}} \frac{4 W_v L}{\pi D^2}$$

and

$$K_2 = \frac{W_v}{W_c} \frac{L Q_{in}}{2\pi \sqrt{R g d^3} \tau_{*c}^{3/2} D^2}.$$

This formulation allows forward modeling of the hydrograph by solving our differential equation as a function of our two dimensionless parameters and the initial conditions. Further, outflow valley erosion history can be computed directly from the solved hydrograph.

**Model Dynamics:** The simplicity of our model allows us to examine the asymptotic dynamics of $T$, which will provide physical insight into the controls on outflow valley erosion. In particular, our model suggests that a competition between lake draining and erosion sets the time-evolution of $T$. Unstable fixed points of $T$ correspond to a transition between a regime where lake-draining outpaces erosion $\left(\frac{dT}{dt} < 0\right)$ and a regime where erosion outpaces lake-draining $\left(\frac{dT}{dt} > 0\right)$. Stable solutions correspond to either runaway erosion scenarios, where erosion continues indefinitely ($T > 1$) or self-arrest scenarios ($T < 1$).

A large lake area, steep outlet valley slopes, and small outlet valley dimensions (low lake-draining parameter, $K_1$) create a large resistance to lake draining. This lake-draining resistance and initial conditions determine the asymptotic behavior of our model in both cases where the inflow parameter may dominate ($K_2 > K_1$) and cases where source inflow doesn't matter ($K_2 < K_1$). In particular, systems with low resistance to lake draining will tend to have outflow hydrographs where the flow depth in the valley tends towards a low, stable value (Figure 2). Whether or not this stable value corresponds to runaway erosion depends on the inflow parameter, $K_2$ (Figure 2). Systems with large resistance to lake draining will have monotonically increasing outflow hydrographs, provided that erosion rates are adequately high (either large enough $K_2$ or large enough initial transport stage; Figure 2). Thus, $K_1$ determines whether or not outflow hydrographs for lake overspill erosion more resemble a brim-full (monotonically increasing hydrograph) or a constant shear stress scenario (e.g. Larsen & Lamb 2016).
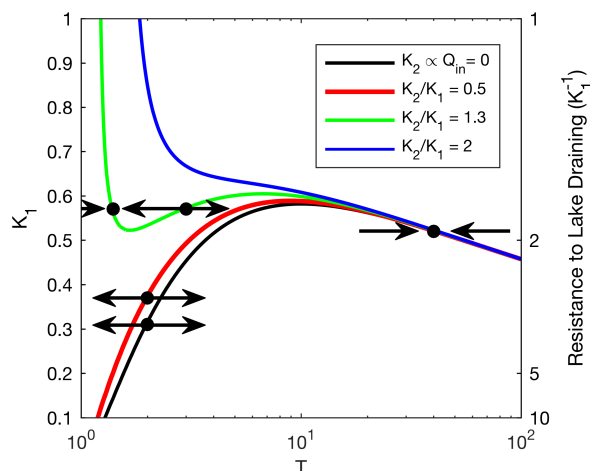


**Figure 2.** Loci of fixed points of transport stage, $T$, as a function of $K_1$, $K_2$. Arrows demonstrate the stability of fixed points. Downward sloping portions of the curves are stable while upward sloping portions are unstable. Stable fixed points at $T < 1$ are not shown. At high transport stages, $K_2$ is negligible and the lake-draining/erosion terms dominate.

**Discussion and Conclusions:** In our model, we have made a number of important assumptions. First, we assumed that $K_1$ and $K_2$ are constant in time. This is unrealistic, as $K_1$ will change due to valley and channel widening/narrowing due to sidewall erosion and terrace abandonment, slope relaxation as relief is consumed, and decreasing effective lake diameter as the valley erodes down (lake walls are not in fact vertical, as assumed in our model and in Figure 2). Further, $K_2$ should change as $Q_{in}$ changes on seasonal timescales. This constant parameter assumption allows for indefinite runaway erosion to occur in the model, which in reality must be shut down eventually by changing parameters (e.g. consumption of relief). Further, we assumed that the channel and valley properties were uniform in space (i.e. no downstream fining of grains by abrasion, no channel concavity, etc.), which allowed us to average over the spatial domain and reduce our model to a 0-D form. While this form is mathematically convenient, we note that the strong sensitivity of our Jezero-relevant simulations to these parameters suggest that understanding the spatial and temporal variations of parameters is important for accurately modeling erosion of lake outflow valleys to their observed depths.

Our model revealed that by adding a small amount of complexity (dynamically determined flow depth), allows the outflow hydrograph to undergo a number of qualitatively distinct behaviors that depends on system parameters and initial conditions. While our model ignores geometric and mechanistic details, inclusion of those details has the potential to introduce more complex classes of behavior. Sensitivity analysis of our model suggests that the transient evolution of the valley geometry (both cross-sectional and long profile) may be important for reconstructing the observed depth of valley erosion. We hypothesize that surveys of open-basin-lake parameters and topography may enable reconstruction of the hydrologic forcing on some, but not all crater-lake overflow valleys.

**References:** [1] Fassett and Head (2008) *Icarus.* [2] Goudge et al. (2015) *Icarus.* [3] Goudge et al. (2016) *Geology.* [4] Goudge and Fassett (2018) *JGR-Planets.* [5] Matsubara et al. (2011) *JGR-Planets.* [6] Goudge and Fassett (2017) *LPSC.* [7] Walder and O'Connor (1997) *Water Resources Research.* [8] Morris et al. (2009) *FLOODsite Report T06-06-03.* [9] Larsen and Lamb (2016) *Nature.*