Non-Steady State Model for Enceladus's Ice Shell. H. S. MacArthur[1] and D. J. Stevenson[2], [1]Seismological Laboratory, California Institute of Technology, Pasadena, Ca, 91125 (hmacarth@caltech.edu) [2]Planetary Sciences, California Institute of Technology, Pasadena, Ca, 91125 (djs@gps.caltech.edu).

Introduction: Libration data show that Enceladus has a global ocean with a mean ice thickness of around 20 km [1]. Earlier gravity data [2] and topography data [3] show that the gravity anomalies at degree 2 and 3 require substantial compensation, meaning that the ice is much thicker at the equator than this average, with a maximum thickness of around 30km. This is a problem because the underside topography of the ice shell will relax by lateral viscous flow. In order to maintain constant thickness with time at that location, there must be continuous freezing of water to compensate for the viscous thinning. (In a global steady state, there is a compensating melting of ice in other regions, especially the poles where tidal heating is presumably stronger). The latent heat release from this freezing must then be accommodated by conduction through the shell, leading to a prediction for the ice thickness. Even in the extreme case of no tidal heating where the ice is thickest, the predicted steady state ice thickness at that location should be ~20 km or less and only weakly sensitive to the ice viscosity [4]. This inconsistency between theory and observation has two possible explanations (1) A much higher viscosity than is usually attributed to water ice at the melting point (around 10^{14} Pa.s), (2) A non-steady state for Enceladus. Although the stresses are very low, existing data do not support the much higher viscosity, especially since that model requires the ice that is flowing to be recently formed by freezing and have small grain size. We accordingly focus here on non-steady state. There are specific models in which the average ice shell thickness varies with time on million year timescales, coupled to variations in the orbital eccentricity. The most recent example of this is a proposed limit cycle [5] but here we focus on a more general approach in which we pose and answer the following question: What are the consequences of a non-steady state thermal profile in the ice shell? If the temperature gradient near the base of the ice is steeper than steady state then the viscous thinning is much reduced, so this motivates consideration of a model in which the shell is currently thinning. In particular, we consider the case where Enceladus was previously in steady-state, but had a much thicker shell. Subsequently, the body was perturbed, perhaps by an increase in the eccentricity, generating an increase in tidal heating

Model: To carry out the model, we solve the 1-D non-steady heat equation,

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2},$$

where κ is the thermal diffusivity of ice, subject to a moving boundary condition at the base of the ice shell: $T(0, h_o - vt) = T_m = 1$ (setting the melting temperature equal to 1 for convenience). The other boundary condition is $T(0,t) = T_s = 0$ (also set to 0 for convenience). The initial condition is given by $T(z,0) = z/h_o$. Here h_0 is the initial thickness of the shell. We consider the case of a thinning shell (positive v). To solve this moving boundary problem, we non-dimensionalize the heat equations and its boundary conditions with lengths measured in terms of h_0, velocities measured in terms of κ/h_o, and time in units of h_o^2/κ. In addition, we remove the moving boundary condition, by introducing the new variable, $\xi = z/(1-vt)$, whence

$$v\xi(1-vt)\frac{\partial T}{\partial \xi} + (1-vt)^2 \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \xi^2},$$

with $T(0,t) = 0$, $T(1,t) = 1$, and $T(\xi,0) = \xi$. We are free to prescribe any value for the velocity v, and initial ice thickness, h_0.

Results: Figure 1 below provides an example computation of the evolution of the temperature profile at equal time intervals of ~ 235 ka for a velocity of 0.71 cm/yr and initial ice thickness of 40 km. (a) corresponds to the evolution of the profile between 0 Ma and ~1 Ma, while (b) corresponds the profiles between ~1 Ma and ~2.3 Ma. The straight line corresponds to the initial linear profile, while the one with the largest curvature corresponds to the temperature profile at the latest time of ~2.3 Ma or, equivalently, an ice thickness of 25 km.

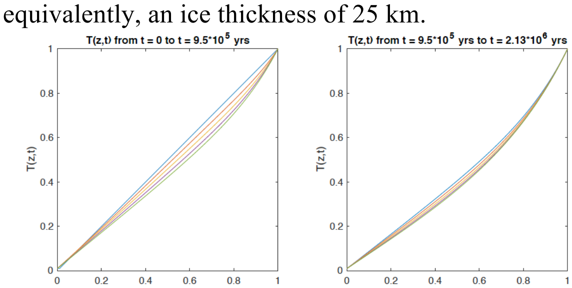

Figure 1: Evolution of the temperature profiles for velocity = 0.71 cm/yr) and initial ice thickness of 40 km. Note that the profile approaches a self-similar value after about 1 Ma. Here the z coordinate is the scaled length ξ.

The separation of the profiles is to both accentuate the rapid steepening that occurs initially and highlight the establishment of self-similarity at about 1 Ma. The effect of this steepening of the temperature gradient at the base of the shell is to concentrate the largest temperatures over a more narrow region, leaving a majority of the ice cold, and rigid. As a result, the total lateral flux of ice decreases until the profile nears self-similarity, at which point the flux approaches a constant.

Having established the evolution of the temperature profile, for a given total, prescribed velocity, we are able solve directly for the velocity component due to viscous flow as well as the component due to freezing/melting. We first assume that ice flow is restricted to the region near the base where the ice remains between melting point, and the 80% of its value (218 K), and approximate the profile between these two points as linear: $T(z) = M(z - z_o) + T_m$, where M is the slope of the profile and $z_o = h_o - vt$, is the total thickness of the shell at time t. This approximation allows us to solve for the viscosity of the ice as a function of depth,

$$\eta(z) \approx \eta(T_m)\exp\left(\frac{AM}{T_m}(z_o - z)\right).$$

Next, in direct analogy with the crustal flow problem studied in [6], the rate of change of thickness of the shell due to flow is found, using the continuity relation:

$$\left.\frac{dD}{dt}\right|_{flow} = \int_{z(0.8T_m)}^{z_o} \frac{\partial v_z}{\partial z} dz = -\int_{z(0.8T_m)}^{z_o} \left(\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{R\sin\theta}\frac{\partial}{\partial\varphi}v_\varphi\right)dz$$

$$\approx -\frac{2}{\eta(T_m)}\left(\frac{T_m}{MA}\right)^3 \frac{\rho g}{R}\left[\frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}h(\theta,\varphi)\right) + \frac{1}{R\sin\theta}\frac{\partial^2}{\partial\varphi^2}h(\theta,\varphi)\right]$$

$$\approx -\frac{2}{\eta(T_m)}\left(\frac{T_m}{MA}\right)^3 \frac{\rho g}{R}\sum_{l=0}^{l=\infty}\sum_{m=-l}^{m=l} l(l+1)h_{lm}Y_{lm}(\theta,\phi).$$

Figure 2 shows the flow velocity and melt/freeze velocity at the equator for the same model parameters in Figure 1. Initially, the flow velocity drops off rapidly due to the large mismatch between thermal diffusion and the rate at which the shell is receding. However, after about 1 Ma, the flow velocity becomes steady. In total, it takes bout 2.3 Ma for the shell to reach an ice thickness of 25 km, close to the desired result for the current shell thickness at the equator.

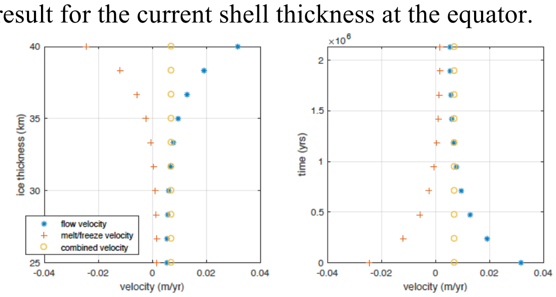

Figure 2: (a) Evolution of the velocities due to flow and freezing/melting as a function of ice thickness. (b) Evolution of the velocities as a function of time.

Discussion/Conclusion: The model we selected for Figure 2 is somewhat arbitrary but illustrates the crucial role of the temperature profile in the ice. If the temperature gradient in the ice nearest the ocean is much steeper than the average profile throughout the ice then the viscous thinning can be reduced by an order of magnitude. An interesting consequence of this result is that the velocity component due to freezing is much reduced, even becoming positive (implying melting) at the point in time where the flow velocity falls below the total prescribed value. It is unclear whether this leads to any conclusion concerning the location of the tidal heating.

Our model does not necessarily address the problem of tidal heating in the ice itself and does not fit with the limit cycle model proposed by Luan and Goldreich (2016), which is based on cyclic, eccentricity driven changes in the amount of tidal heating in the shell. Also, their model suggests Enceladus is currently thickening rather than thinning. We could have started with a thinner shell, and prescribed a negative velocity (thickening) but the ice would readily flow away to those regions of low topography, requiring an unreasonably large velocity component due to freezing to compensate.

Our model is consistent with a majority of the tidal heating being concentrated in the core [7], as all of the heat would eventually be conducted to the shell via the ocean. We reserve caution in justifying our model on core heating alone as tidal heating in the shell is likely to be significant. Despite these apparent difficulties in mechanically substantiating our model, it is clear that it does permit solutions where the present day ice thickness is closer to its true value than the steady-state case.

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