

# CONSTRAINTS ON MODELS OF THE INTERIOR STRUCTURE OF MERCURY FROM MEASUREMENTS OF ITS MOMENT OF INERTIA AND TIDAL RESPONSE.

Sander Goossens<sup>1,2</sup>, Wade G. Henning<sup>3,2</sup>, Joe P. Renaud<sup>4</sup>, Antonio Genova<sup>5</sup>. <sup>1</sup>Center for Research and Exploration in Space Science and Technology, University of Maryland, Baltimore County, 1000 Hilltop Circle, Baltimore MD, USA (email: [sander.j.goossens@nasa.gov](mailto:sander.j.goossens@nasa.gov)), <sup>2</sup>NASA Goddard Space Flight Center, Code 698, 8800 Greenbelt Road, Greenbelt, MD 20771, USA, <sup>3</sup>University of Maryland, College Park, Department of Astronomy, College Park, MD 20742, USA, <sup>4</sup>George Mason University, Department of Physics and Astronomy, 4400 University Drive, Fairfax, Virginia 22030, USA, <sup>5</sup>Dipartimento di Ingegneria Meccanica e Aerospaziale, Sapienza Università di Roma, Via Eudossiana 18, 00184 Rome, Italy,

**Introduction:** Knowledge of the interior structure of a planet is important: it reflects the planet's formation and subsequent evolution, including its differentiation into layers such as a core, mantle, and crust. In the absence of seismic data, gravity provides important insights into a planet's interior, as the gravity field depends on a world's density structure.

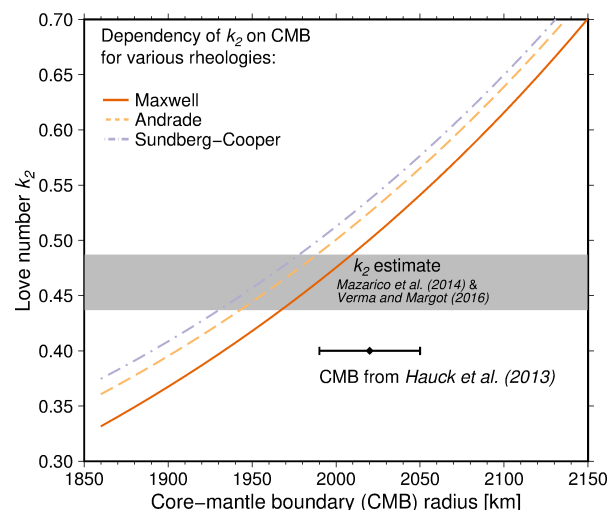
Mercury is difficult to observe both astrometrically and by spacecraft exploration, because of its proximity to the Sun. It can be considered an end-member of formation processes in the solar system because of its location and its high bulk density [1]. The MESSENGER [1] mission was the first spacecraft to orbit the planet Mercury. Its mission accomplishments included, among many others, the first detailed maps of the gravity [3,4] of Mercury. Combined with measurements of Mercury's spin state, this resulted in a measurement of the planet's moment of inertia (MoI) [5], which describes the radial density distribution in a planet about a given axis. In turn, these results were used to greatly improve determinations of the interior structure of Mercury [6,7], providing constraints on the radius of the liquid core, and the densities of the core and silicate layer. Mercury's tidal response has also been used to investigate models of its interior [8].

In order to determine Mercury's MoI from spin and gravity data, Mercury is assumed to be in the Cassini state I [9]. Measurements of Mercury's spin state have shown that, within error bars, the planet is indeed in this Cassini state. Yet only a recent analysis of MESSENGER gravity has resulted in a measurement of Mercury's obliquity that unambiguously satisfies the Cassini state I [10]. The measured MoI with this new obliquity is lower than the previous value, indicating a larger degree of internal differentiation in the planet than assumed before. Extensive modeling of Mercury's interior structure indicates that a solid inner core for Mercury is likely [10]. We now extend this recent study to include Mercury's tidal response, as described by the potential degree 2 Love number  $k_2$ , as a constraint in our estimation of Mercury's interior structure.

**Data and Methods:** We use data from the recent gravity model using all MESSENGER tracking data, which resulted in the measurement of Mercury's

obliquity being unambiguously in the Cassini state I [10]. The data we use include the newly derived MoI values, consisting of the normalized polar moment of inertia and the fractional polar moment, which is the crust and mantle part divided by the full polar moment, as well as a newly derived value for  $k_2$ . We also use the planet's bulk density as a constraint.

We model Mercury's interior structure as follows: using hydrostatic assumptions, we numerically integrate the differential equations for pressure, gravity and temperature, using the boundary condition that pressure is zero at the surface. We set the reference radius to 2440 km and assume a spherically symmetric planet, divided into layers of 1 km thick. We assume a conducting mantle with constant heat production rates in the mantle and crust [8], and we vary the temperature at the core-mantle boundary (CMB). We assume an adiabatic profile for the outer core, and an inner core that is isothermal. To relate temperature and pressure to density, we use the third order Birch-Murnaghan Equation of State (EoS), which has been used before extensively for Mercury [6,7].



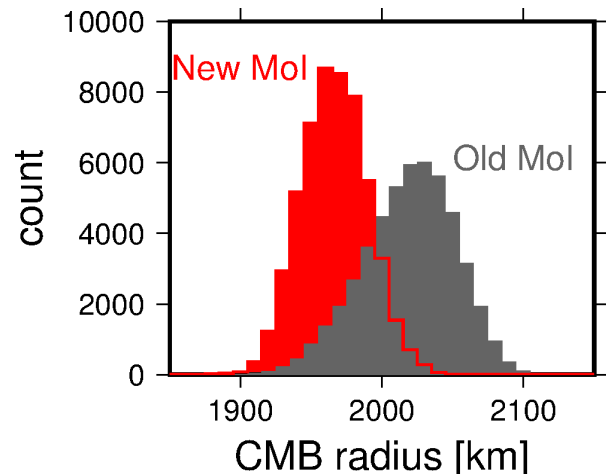
**Figure 1** The location of the CMB can be constrained by  $k_2$  as there is a strong dependency between the two. Here we show how  $k_2$  varies with the CMB radius for a given interior structure, using various rheologies when computing the planet's tidal response.

For each self-consistent interior structure model, we then compute the polar and fractional moments, the bulk density, and the tidal response as expressed by  $k_2$ . We compute  $k_2$  from Mercury's multi-layer solid tidal response, using the software: ALMA [11]. We have updated this software to include several rheologies: the Maxwell, Andrade, and the recent Sundberg-Cooper rheology [12]. In Figure 1 we show the influence of different rheologies on  $k_2$ , for a given interior structure model where we only vary the CMB radius.

In order to determine a likely interior structure model that fits all four constraints (two MoI values, the bulk density, and  $k_2$ ), we have adopted a Markov-Chain Monte-Carlo (MCMC) method using the Metropolis-Hastings [13,14] scheme. Our estimated parameters are: the inner core radius, outer core (CMB) radius, crustal thickness, mantle density, crust density, temperature at the CMB, and weight fractions of light elements in the core that determine core density (we use either an FeS or FeSi core). To characterize the tidal response, we also include as parameters: the unrelaxed rigidities for the crust and mantle, a silicate reference viscosity, and grain size. This results in 12 parameters with only 4 measurements, and the determination of a likely interior structure model is thus inherently non-unique. It is however possible to constrain parameters of interest subject to available constraints using Bayesian probability theory, which has been done extensively for the Moon, Mars, and Mercury. To determine likely interior structure models for Mercury, we run multiple chains with randomized starting conditions. Each chain will advance by selecting changes to the parameters from a proposal distribution, for which we will use a Gaussian distribution. We also assume a Gaussian distribution for the four target parameters, using their reported central values and standard deviation. We will then map out the probability density functions of the parameters of interest.

**Results:** The Love number  $k_2$  is mostly sensitive to the size of the mantle and processes therein, and indeed we find updated constraints for the CMB radius. All of our results that include  $k_2$  as an additional constraint are furthermore fully consistent with the recent interior model evaluations using the lower MoI value [10]. This includes the indications for the existence of a solid inner core, which remain unchanged (we note that  $k_2$  is not sensitive to the presence of an inner core [8]). These latest results also found a smaller CMB radius than inferred from the previous MoI values, and our results confirm that the updated Love number is consistent with such a model with smaller CMB radius. In Figure 2, we show the result of the CMB values obtained from our MCMC analysis. We compare the new CMB radius with that obtained from the same MCMC analysis which uses the previous MoI values instead. Those

results are consistent with earlier investigations by several authors [6,7].



**Figure 2** The distribution of CMB radius values, as a result from our implemented MCMC algorithm, using the new and old MoI values. The new MoI value results in a lower CMB radius value than previously found. Results here also include  $k_2$  as a constraint.

The current error bounds on  $k_2$  are such that our results cannot distinguish between the different rheologies as shown in Figure 1. All rheologies map out the target distribution equally well, and while there are some differences in the obtained histograms for CMB temperature, they are not significantly different. Future improvements in the value of  $k_2$  may be better able to assess the influence of different rheologies.

We will also present the results for additional parameters such as mantle density, core pressure, core density, and CMB and inner core temperatures.

**References:** [1] Chapman, C.R. (1988), in *Mercury*, Eds. Vilas, F. *et al.*, pp. 1-23. [2] Solomon S.C. *et al.* (2007), *Sp. Sci. Rev.*, 113, pp. 3-39. [3] Smith D.E. *et al.*, *Science*, 336, pp. 214-217. [4] Mazarico E. *et al.* (2014), *J. Geophys. Res. Planets*, 119, pp. 2417-2436. [5] Margot, J.L. *et al.* (2012), *J. Geophys. Res. Planets*, 117, E00L09. [6] Hauck, S.A. *et al.* (2013), *J. Geophys. Res. Planets*, 118, pp. 1204-1220. [7] Knibbe, J.S and van Westrenen, W. (2015), *J. Geophys. Res. Planets*, 120, pp. 1904-1923. [8] Padovan, S. *et al.* (2014), *J. Geophys. Res. Planets*, 119, pp. 850-866. [9] Peale, S.J. *et al.* (2002), *Meteorit. Planet. Sci.*, 37, pp. 1269-1283. [10] Genova A. *et al.* (2019), submitted. [11] Spada, G. (2008), *Comput. And Geosci.*, 34, pp. 667-687. [12] Renaud, J.P. and Henning, W.G. (2018), *Astrophys. J.*, 857(2), 98. [13] Metropolis, N. *et al.* (1953), *J. Chem. Phys.*, 21, pp. 1087-1092. [14] Hastings, W. (1970), *Biometrika*, 57, pp. 97-109.