CONVECTION IN TITAN LAKES: FLUX-DRIVEN WITH TIME-DEPENDENT UPPER BOUNDARY CONDITION . D. G. Korycansky, *CODEP, Department of Earth and Planetary Sciences, University of California, Santa Cruz CA 95064* , O. Umurhan, *SETI Institue, Mountain View, California USA 94043* .

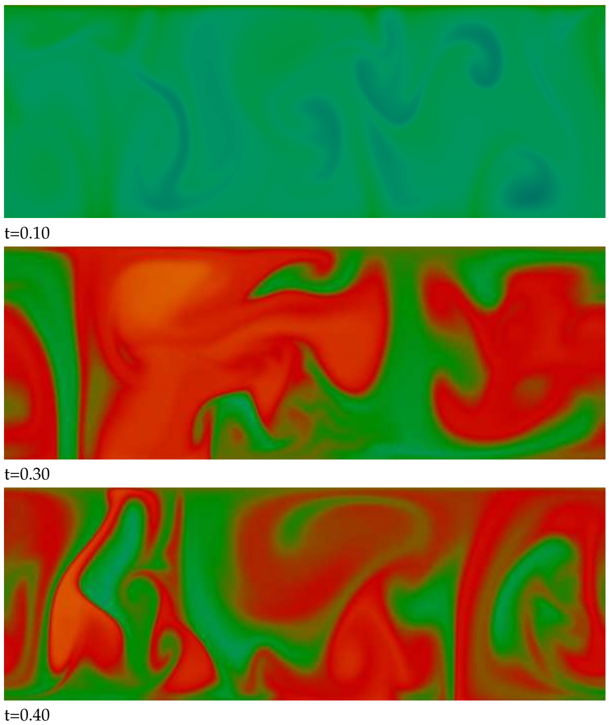

Figure 1: Three timesteps of a convection calculation with $Ra = 10^7$, $Pr = 1$, with bottom boundary condition $dF/dz = 1$, top boundary condition $T_t = 0.5\Delta T_0[1+\sin(2\pi t/P)]$, and $P = 0.01$.

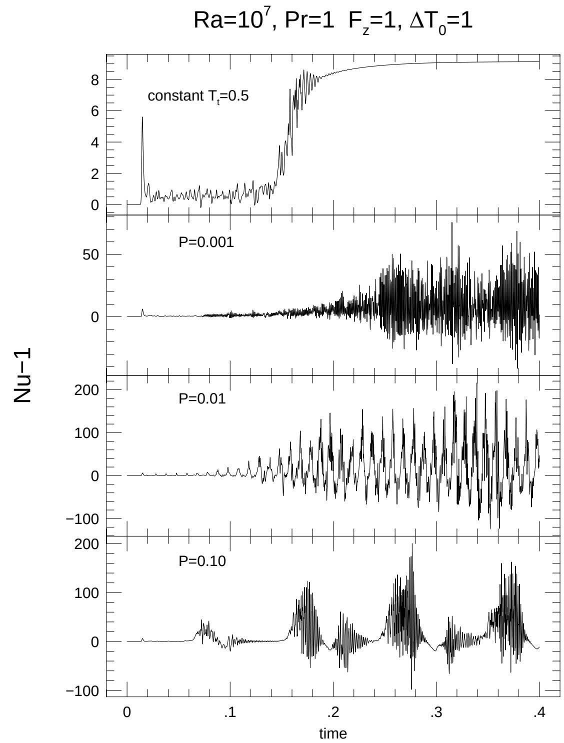

Figure 2: Plots of Nusselt number $Nu-1$ vs. time for four different convection models differing the period P of the thermal boundary condition at the top of the grid. (The top panel has a constant temperature $T_t = 0.5$ at its top boundary.) Calculations were done on a 512×182 with $Ra = 10^7$ and $Pr = 1$.

Introduction

One of the most striking aspects of Titan's surface is the presence of large lakes and seas composed of liquid methane and ethane. The volume of these liquid bodies is comparable to the largest lakes on Earth.

Based on the idea that solar insolation on the 16-day diurnal cycle of Titan's orbit around Saturn, we are investigating the character of possible convection in Titan's lakes. We assume that there is a signficant thermal flux from Titan's interior that is capable of driving convection, coupled with diurnal modulation of the temperature at the top surface. At this stage we are investigating idealized convection models with the idea of understanding the effects of the temporal boundary modulation on overall heat transport in a convective situation.

Convection modeling

We solve the equations for two-dimensional convection in the Boussinesq approximation (Tritton 1988) using a vorticity-streamfunction formulation:

$$\frac{\partial \omega}{\partial t} + \boldsymbol{u}\cdot\nabla\omega = -RaPr\frac{\partial T}{\partial x} + Pr\nabla^2\omega,$$
$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T = \nabla^2 T$$
$$\omega = -\nabla^2\psi, \quad \boldsymbol{u} = \nabla\times\psi \qquad (1)$$

The equations have been non-dimensionalized in terms of the domain depth L_z, a nominal temperature difference $\Delta T_d = L_z dT_d/dz$, and the diffusion time $t_d = L_z^2/\kappa$. The temperature is T, the vorticity is $\omega = \nabla\times u$, the streamfunction is ψ with the velocity $\boldsymbol{u} = (u,w)$ derived from the curl of ψ.

The parameters determining the strength of the buoyancy and viscous forces are the Rayleigh number $Ra = g\alpha\Delta T_d L_z^3/\nu\kappa$ and the Prandtl number $Pr = \nu/\kappa$. The bottom boundary for temperature condition is given by the (non-dimensional) flux $F_z = dT/dz$ and for the top we have a time-dependent temperature $T_t = 0.5\Delta T_0[1+\sin(2\pi t/P)]$ so that the temperature has a sinusoidal time-dependence with period P and amplitude T_0. We have free-slip velocity boundary conditions on top and bottom ($\psi = 0$, $\partial\omega/\partial z = 0$). All quantities are horizontally periodic, and we Fourier-transform the variables in x and use finite differences in z for the solution of Poisson's equation for the streamfunction. Quantities are transformed back to real space in x for the finite difference solution (using an Arakawa scheme for the non-linear terms, Arakawa and Lamb 1981) of the vorticity and temperature equations, with a second-order Adams-Bashforth scheme for time advancement. The calculations we show in this abstract were done with $Ra = 10^7$ and $Pr = 1$ on a 512×182 grid with constant bottom heat flux and time-dependent temperature at the top as described. The horizontal aspect ratio of the grid is $L_x/L_z = 2.828$. A quantity of interest is the Nusselt number Nu, which gives the vertical heat transport $< w\partial T/\partial z >$ normalized to a nominal diffusive heat flux $\kappa dT_d/dz$, where $T_d = 1-z$ is a nominal convectively stable linear temperature profile.

Some preliminary results

The initial condition is $T = 0$ throughout the domain. Convection is driven by the bottom heat flux dF/dz. The temperature modulation at the top is sufficiently large to shut down convection, although if the period P is long enough, the continued input flux would re-establish convection that would follow the (slowly-changing) top boundary condition. We are interested in conditions where the temperature at the top is changing rapidly enough to influence heat transport through the domain.

The figures show some results for runs with the flux bottom boundary condition and different modulation periods for the top boundary temperature. Figure 1 shows the temperature field at (non-dimensional) times $t = 0.1$, 0.3, and 0.4 for the case in which the modulation period $P = 0.01$. Figure 2 shows $Nu-1$ for three different cases with different top-boundary temperature oscillation periods P. For the shortest period P, the modulation controls the overall behavior and the heat transfer is limited. For longer periods, the convective activity more closely follows the underlying temperature field as it diffuses from the upper boundary; convection increases when the gradient of temperature field steepens.

Acknowledgments

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References

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