

TIDAL DISSIPATION IN RUBBLE-PILE ASTEROIDS AND ICY BODIES. F. Nimmo¹, I. Matsuyama²¹Dept. Earth and Planetary Sciences, University of California Santa Cruz, Santa Cruz, CA 95064, USA ² Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721, USA

Summary: We derive a simple theory describing tidal dissipation in the regolith layer of rubble-pile binary asteroids [1]. The theory agrees with inferred tidal dissipation rates if the regolith thickness is independent of body size. Applications to asteroid Bennu and KBO Ultima Thule are discussed.

Introduction: The tidal response of a body provides information on its internal structure and mechanical properties [2]. The rate of tidal dissipation depends on k_2/Q or alternatively $1/\mu Q$, where k_2 is the tidal Love number (giving the response amplitude), Q is the dissipation factor (giving the response phase) and μ is the effective rigidity [3,4].

Observations: Jacobson and Scheeres [5] used binary asteroids to determine k_2/Q , by assuming that the observed semi-major axis was a result of equilibrium between tidal dissipation and the binary YORP effect. An alternative [6] is to neglect the second effect and instead assume a system age; this yields a higher (less dissipative) Q/k_2 value. A recent astrometric study of 1996FG₃ [7] showed no semi-major axis evolution and thus supports the equilibrium assumption.

Theory: An important theoretical treatment by [8] showed that yielding in rubble-pile asteroids results in an effective rigidity much less than that of a monolithic body, and predicts that k_2 scales with primary radius R .

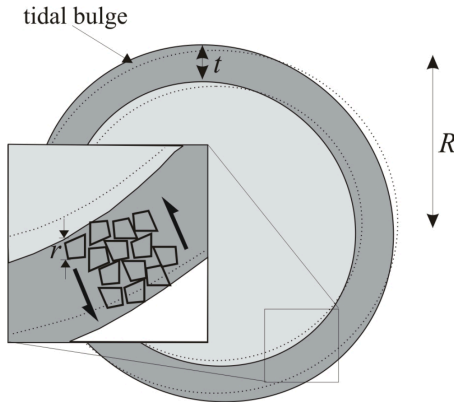


Figure 1: Sketch of the geometry of the problem. Tidal deformation is represented by the departure of the solid surface from the mean shape (dashed lines) and results in shear strains (indicated by half-arrows).

We focus on tidal dissipation in a regolith layer of thickness t (Figure 1). The motion of the tidal bulge relative to the surface results in shearing motion of individual regolith blocks. On a single block face the frictional dissipation rate depends on the shear stress, surface area and the sliding velocity, and is given by

$$\dot{E}_f \sim (\rho g t) f r^2 u \Omega_p \quad (1)$$

where ρ is the density, g the surface acceleration, r the block size, u the relative displacement between neighbouring faces, Ω_p the spin rate and f the friction coefficient. The displacement depends on the tidal strain ϵ and the block dimensions and may be written

$$u \sim \epsilon r \sim r \left[\frac{H}{R} \right] \sim r h_2 \left[\frac{q}{1+q} \frac{n^2}{G\rho} \right] \quad (2)$$

where H is the amplitude of the tidal bulge, h_2 is the tidal displacement Love number, q is the mass ratio between secondary and primary, n is the mean motion of the secondary and G is the gravitational constant.

Combining equations (1) and (2) we derive the total dissipation rate in the regolith layer of

$$\dot{E}_f \sim N f h_2 M q n^2 \Omega_p t^2 \quad (3)$$

Here N (~ 3 for a roughly cubic element) is the number of faces per element, m is the mass of the secondary and we have dropped the $(1+q)$ term which is generally close to unity. The t^2 term arises because increasing t increases the overburden pressure and the total dissipative volume. As expected, dissipation increases with friction coefficient, forcing frequency and displacement (h_2); it is also independent of the element size r , as long as $r < t$.

Equation (3) may be compared with the conventional expression for tidal dissipation in a non-synchronous body [9] to derive an effective Q , given by

$$Q_{eff} \sim \left[\frac{q n^2}{G\rho} \right] \frac{1}{N f} \left(\frac{R}{t} \right)^2 \sim 3 \times 10^{-3} \left(\frac{q n^2}{3 \times 10^{-10} \text{ s}^{-2}} \right) \left(\frac{2 \text{ g/cc}}{\rho} \right) \left(\frac{R}{t} \right)^2$$

where here we have assumed that $h_2 \approx k_2$.

This result may then be combined with the prediction of [8] to derive equation (4):

$$\frac{Q}{k_2} \sim 3 \times 10^5 \left(\frac{R}{1 \text{ km}} \right) \left(\frac{q n^2}{3 \times 10^{-10} \text{ s}^{-2}} \right) \left(\frac{2 \text{ g/cc}}{\rho} \right) \left(\frac{30 \text{ m}}{t} \right)^2$$

The same result can also be used to predict the quantity μQ , here given in SI units:

$$\mu Q \sim 10^8 \left(\frac{R}{1 \text{ km}} \right)^3 \left(\frac{q n^2}{3 \times 10^{-10}} \right) \left(\frac{30 \text{ m}}{t} \right)^2 \quad (5)$$

Comparison with observations: We use the approach of [5] but with an expanded catalogue of asteroid binaries, from [10] and using a BYORP parameter $B=10^{-2}$ [7]. The inferred Q/k_2 as function of body radius is shown in Fig 2. As noted by [5], the inferred Q/k_2 scales roughly with R (or $R^{1.5}$), which is opposite to the prediction of [8] if Q is constant. In contrast, the observations are consistent with equation (4) if t is constant, or decreases slightly with radius. Furthermore, the dependence on qn^2 , shown by colours in Figure 2, is also approximately consistent with equation (4)

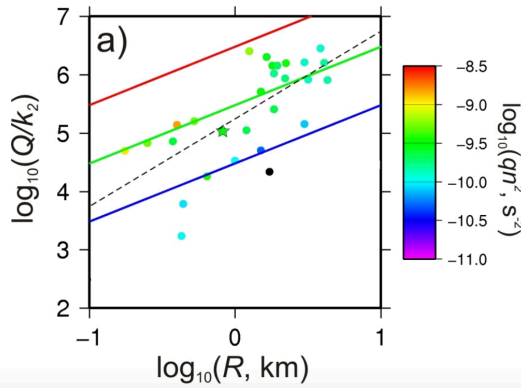


Figure 2: Dots are data plotted from [1] taking $B = 10^{-2}$ (see text); colour indicates the quantity qn^2 . Star is (175706) 1996 FG₃ [7]. Dashed line shows least-squares fit to the data, with a gradient of 1.51. Coloured lines use equation 4 with three different values of qn^2 ($10^{-8.5}$, $10^{-9.5}$, $10^{-10.5} \text{ s}^{-2}$) and $t=30 \text{ m}$.

A further observation of relevance is a study of tumbling asteroids by [11]. These authors argue that the damping timescale for such tumbling is approximately independent of radius. For this to be the case, μQ would need to scale as R^2 . Our simple analysis (equation 5) predicts an R^3 dependence; thus, there is qualitative but not quantitative agreement.

Regolith Thickness: Figure 2 suggests that the regolith thickness $t \sim 30 \text{ m}$, independent of body radius. Only rather scanty estimates of regolith thickness are available: a few metres or more on Itokawa ($R=0.17 \text{ km}$) [12]; 30-200 m on Gaspra ($R=6.1 \text{ km}$) [13]; up to a few tens of metres on Eros ($R=8.4 \text{ km}$) [14]; 100-200 m on Phobos ($R=11.3 \text{ km}$) [15]; $\sim 50 \text{ m}$ on Ida ($R=15.7 \text{ km}$) [16]. Based on these results it certainly seems as if regolith thickness varies only rather weakly with radius, and a $\sim 30 \text{ m}$ thickness would be hard to rule out. A theoretical study by [17] argued that the expected regolith thickness is tens of metres, and should decrease slightly with R . This prediction is in good agreement with our results.

Application to asteroid Bennu: For an isolated rubble-pile asteroid undergoing a wobble of amplitude α the dissipation rate within a regolith layer is

$$\dot{E}_f \sim N f h_2 M \alpha \Omega_w \Omega_p^2 t^2 \quad (6)$$

which is analogous to equation (3) with the strain rate determined by the rotational bulge, the wobble angular frequency Ω_w and amplitude α . The quantity Ω_w is smaller than Ω_p by a factor of $h_2 \Omega_p^2 / G \rho$ [18]. Using the total wobble energy from [18], the damping timescale may be written

$$\tau \sim \left(\frac{R}{t}\right)^2 \frac{\alpha}{N f h_2 \Omega_p} \quad (7)$$

The damping timescale decreases with increasing friction or bulge amplitude (h_2) as expected. If Bennu is a rubble-pile asteroid, $R \sim 300 \text{ m}$ implies $h_2 \sim 3 \times 10^{-6}$ [8]. For a regolith thickness of 30 m the damping timescale is then $\sim 10^6$ rotation periods, or $\sim 10^3$ years. This is shorter than conventional estimates [18], primarily because h_2 is larger than the equivalent calculation for a monolithic body.

Application to Ultima Thule: Relatively little attention has been paid to tidal dissipation in KBO binary pairs [19]. If Ultima Thule ($R \approx 10 \text{ km}$) is a contact binary, one possible contributing mechanism is tidal dissipation: if the secondary starts inside the synchronous point (as with Phobos), or is retrograde (as with Triton), it will evolve inwards, and the primary will spin up. Additional mechanisms draining angular momentum, such as dynamical friction or impacts, may also have occurred.

Although there is no particular reason to expect regolith thickness to be the same on KBOs as on asteroids, application of equation (4) with $t=30 \text{ m}$ would yield $Q/k_2 \sim 10^8$ just prior to contact, and a lower value (more dissipative) at greater separations. Conventional calculations for a monolithic ice body with $Q=100$ would yield $Q/k_2 \sim 3 \times 10^8$, resulting in tidal evolution at least a few times slower than in the dissipative regolith case.

References: [1] Nimmo & Matsuyama, *Icarus* 2019 [2] Moore & Schubert, *Icarus* 2000 [3] Munk & MacDonald, *The rotation of the Earth*, 1960 [4] Efroimsky & Makarov, *Ap.J.* 2013 [5] Jacobson & Scheeres, *Ap. J. L.* 2011 [6] Taylor & Margot, *Icarus* 2011 [7] Scheirich et al., *Icarus* 2015 [8] Goldreich & Sari, *Ap. J.* 2009 [9] Murray & Dermott, *Solar System Dynamics*, 1999 [10] Pravec et al., *Icarus* 2016 [11] Pravec et al., *Icarus* 2014 [12] Barnouin-Jha et al., *Icarus* 2008 [13] Veverka et al., *Icarus* 1994 [14] Prockter et al., *Icarus* 2002 [15] Thomas et al., *JGR* 1979 [16] Sullivan et al., *Icarus* 1996 [17] Langevin & Maurette, *LPSC* 1980 [18] Burns & Safronov, *MNRAS*, 1973 [19] Grundy et al., *Icarus* 2011.