

IMPACT VELOCITY BETWEEN PARTICLES IN SATURN'S RINGS. H. Kawamura¹, K. Ohtsuki¹, N. Hirata¹, and H. Daisaka², ¹Department of Planetology, Kobe University, Kobe, 675-8501, Japan, ²Graduate School of Commerce and Management, Hitotsubashi University, Tokyo 186-8601, Japan.

Introduction: Saturn's rings are mostly composed of water ice. Particles in the rings experience frequent mutual collisions, which govern dynamical evolution of the rings. Since outcomes of collisions are expected to vary depending on impact velocity, it is important to evaluate impact velocities between particles in the rings.

Spacecraft and ground-based observations show that particles in Saturn's rings have a power-law distribution roughly between centimeter and ten meters, but sub-centimeter particles are lacking [1-3]. As an explanation for the paucity of sub-centimeter particles, sticking of such small particles onto the surface of large particles has been proposed [4,5]. Since smaller particles have larger surface-to-volume ratios, the cohesive force is more effective for small particles. Therefore, when small particles hit the surface of large particles, they can stick to the surface if the size of the small particle is smaller than a critical size. This critical size depends on impact velocity. A recent study examined sticking and collisional release of small particles on the surface of large particles, and showed that the amount of free small particles increases with increasing velocity dispersion in the rings [5]. In this study, impact velocity was estimated from an analytic estimate based on N-body simulation of the velocity dispersion in the rings of equal-sized particles. However, impact velocity can be smaller than the velocity dispersion when particles move coherently forming gravitational wakes in dense rings [6,7]. Furthermore, analytic estimate is difficult when size distribution of particles is taken into account.

In the present work, we perform local N-body simulation of Saturn's main rings and examine impact velocities between particles in each of the three main rings. We investigate dependence on various parameters, such as the restitution coefficient, distance from the planet, and optical depth of the ring.

Model: We adopt the method of local N-body simulation [6,7] and use a code based on our previous work [8,9]. We erect a rotating Cartesian coordinate system with the origin at the center of the square simulation cell that moves on a circular orbit. The x -axis points radially outward, the y -axis points in the direction of the orbital motion, and the z -axis is normal to the equatorial plane. We directly calculate the gravitational forces between particles using GRAPE-9, and orbits of particles are integrated with the second-order leap-frog method.

When collisions between particles are detected, velocity changes are calculated based on the hard-sphere model, assuming that particles are smooth spheres. As

for the normal restitution coefficient, we examined the following four cases: (i) A constant value of 0.1; (ii) the velocity-dependent restitution coefficient based on laboratory experiments [10] with the reference velocity $v_c = 1 \text{ cm s}^{-1}$ ($\equiv v_B$) [6,10]. (iii) Same as (ii) but more dissipative case with $v_c = 0.25v_B$; and (iv) Same as (ii) but more elastic case with $v_c = 5v_B$. In order to avoid the influence of initial conditions, calculations of velocity dispersions and the impact velocities were carried out using numerical data taken after the system has reached a quasi-steady state.

Results: Figure 1 shows the distribution of the impact velocities in the case of a ring of equal-sized particles with radius of 1m. The radial location and the surface density were chosen so that they are similar to those for the A ring, i.e., the semi-major axis is $1.28 \times 10^{10} \text{ cm}$ and the surface density is 43 g cm^{-2} . The material density of particles was assumed to be 0.4 g cm^{-3} . We found that most collisions take place at low velocities, with impact velocity less than 0.1 cm s^{-1} , which is significantly smaller than the velocity dispersion that can be estimated analytically. This is because particles move coherently in gravitational wakes, as shown by previous works [6,7]. On the other hand, we also found that infrequent high-velocity collisions also take place, with impact velocity higher than 0.5 cm s^{-1} . Such high-velocity impacts occur when adjacent wakes collide with each other. In fact, we analytically calculate Kepler-shear velocity between adjacent wakes assuming that their radial separation is approximately given by the critical wavelength for the gravitational instability, and found that the above high-impact velocity can be roughly reproduced by this estimate. We also examined the dependence of the maximum impact velocity on restitution coefficient, and found that the maximum value becomes smaller for more elastic cases, because the formation of wakes becomes difficult when collisions are more elastic.

Figure 2 shows mean impact velocity and radial velocity dispersion as a function of particle size in the case of particle size distribution. The parameters of this simulation were those for the A ring, and the particles are assumed to have a power-law size distribution from 30cm to 3m. In the case of the most elastic particles (i.e., case (iv)), we find that the size-dependence of the velocity dispersion is notable, because in this case large particles tend to strongly perturb the motion of small particles. On the other hand, when collisions are more dissipative, size-dependence of the velocity dispersion

becomes significantly weak, because particles tend to move together in gravitational wakes. As a result, we find a similar tendency in the results of impact velocity. That is, the size-dependence of impact velocity is rather weak in such dense rings when collisions are sufficiently dissipative, because particles tend to move coherently in gravitational wakes.

Our results show that most collisions in dense rings take place with impact velocity significantly smaller than the velocity dispersion. We also found that infrequent high-velocity impacts also take place, which can be explained by impacts between adjacent wakes. These results are important in estimating the size of the smallest particles in Saturn's rings, because it depends on impact velocity as mentioned above.

Acknowledgments: This work was supported by JSPS KAKENHI Nos. 15H03716, 16H04041, and 18K11334. Part of the numerical simulations were performed using computer systems at the National Astronomical Observatory of Japan.

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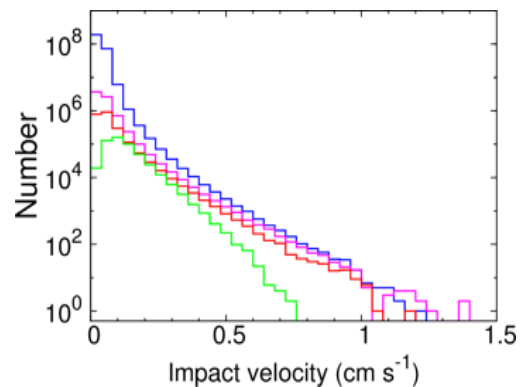


Figure 1: Distribution of impact velocities for a ring with radial location and surface density corresponding to the A ring. Particles are assumed to have the same size in this case. The four lines represent the cases with different restitution laws: (i) Constant normal restitution coefficient, 0.1 (blue); (ii) the velocity-dependent restitution coefficient based on Bridges et al.1984 (red); (iii) Similar to (ii) but more dissipative case (magenta); (iv) Similar to (ii) but more elastic case (green).

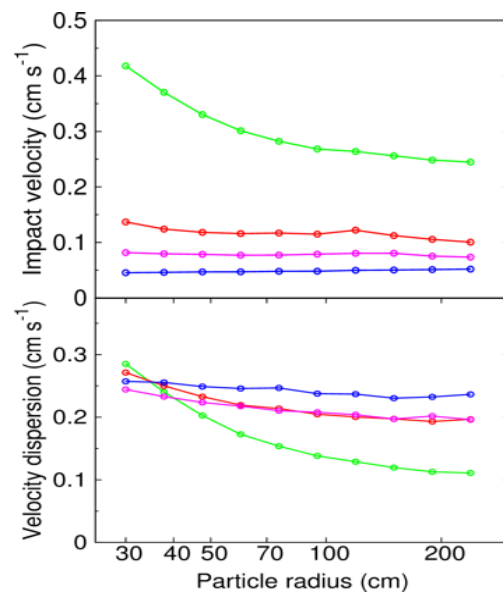


Figure 2: Mean impact velocity and radial velocity dispersion as a function of particle size in the case of particle size distribution. We assumed a power-law size distribution with an index of -3 for particle radii between 30cm and 3m. The radial location and surface density correspond to those of the A ring. The colors represent different restitution coefficient laws mentioned in the caption of Figure 1.