POLARIZED LIGHT SCATTERED FROM ASTEROID SURFACES. VIII. INTERRELATION BETWEEN THE INVERSION ANGLE OF POLARIZATION DEGREE AND THE BRIGHTNESS OPPOSITION
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Since a theory of the polarization of light scattered from rough planetary surface has not yet been developed, a comprehensive study of the properties of polarized radiation from planets and asteroids remains an important task. Therefore, it is not surprising that many researchers pay attention to finding relationships between the polarimetric and photometric properties of asteroids [e.g., 1-3]. Not long ago [4-6], we have shown that for the asteroids of $\mathrm{C}, \mathrm{M}, \mathrm{S}$, and E types such characteristics of the negative polarization branch as its amplitude $\left|P_{\text {min }}\right|$ and the polarimetric slope $h$ as well as the corresponding phase angle $\alpha_{\text {min }}$ and inversion angle $\alpha_{i}$ closely correlate with the phase coefficient of the photometric function, $b$, and the photometric roughness of the asteroid surfaces, $c$.

Another parameter of asteroid photometric function, named the amplitude of opposition surge in brightness, $a_{B O E}$, correlates with the geometric albedo of asteroid surfaces. This $\cap$-like dependence of $a_{B O E}$ on asteroid albedos was found in [7] and discussed in [6, 8]. In turn, we undertook a search for relationships between $a_{B O E}$ and the parameters of negative polarization branch, and this hunting was successful.

Initial data were taken from the polarimetric and photometric catalogues of asteroids available at NASA PDS [9, 10]. In order to calculate the parameters of asteroid photometric function (magnitudes in the V bandpass depending on phase angle $\alpha$ ) we used the following approximating formula, which is correct at phase angles $\sim 30^{\circ}$ and less [7]:
$\Delta V(\alpha)=\left[-a_{B O E} /(1+\alpha)\right]+b \times \alpha$.
We approximated the polarimetric function of asteroids (i.e. the phase dependence of polarization degree) in the visible spectral range with the three-parameter equation adopted for the small phase angles [6]:
$P(\alpha)=\frac{h\left(1-e^{-m \alpha}\right)\left(\alpha-\alpha_{i}\right)(\alpha-\pi)}{\left(1-e^{-m \alpha_{i}}\right)\left(\alpha_{i}-\pi\right)}$.
For asteroids with the known polarimetric and photometric parameters we found correlation between the amplitude of brightness opposition effect $a_{\text {BOE }}$ and the value of inversion angle $\alpha_{i}$ (see Fig. 1). This dependence is statistically significant with a correlation coefficient of 0.78 .


Fig. 1. The BOE amplitude of asteroids against the inversion angle of their phase-dependent polarization curves.


Fig. 2. The angle of the BOE beginning against the inversion angle of asteroid polarimetric function.

It is unclear now what characteristics of lightdiffusing surfaces specify the correlation between $a_{B O E}$ and $\alpha_{i}$. We can say only the BOE amplitude defines the phase angle $\alpha_{B O E}$ at which the nonlinear increase of a magnitude-phase dependence begins. Generally speaking, the derivative $\mathrm{d} V(\alpha) / \mathrm{d} \alpha=\left[a_{B O E} /(1+\alpha)^{2}\right]+b$ tends to $b$ if $\alpha$ tends to infinity. In other words, the value of $\alpha_{B O E}$ should be much greater than unity. Since $\mathrm{d} V(\alpha) / \mathrm{d} \alpha=b+\Delta b$ we can define a boundary, where the linear magnitude-phase dependence is replaced by a non-linear one. The phase coefficient is determined with some error $\sigma_{b}$, therefore, $\Delta b$ cannot be less then $\sigma_{b}$. Taking into account the 3 -sigma rule, we can assume that $\Delta b \approx 3 \sigma_{b}$. Thereby we have
$\alpha_{B O E}=\left(a_{B O E} / 3 \sigma_{b}\right)^{1 / 2}-1$, where $\sigma_{b} \approx 0.001$.

This formula establishes linkage between the angle of the BOE beginning and BOE amplitude. Bearing in mind this fact as well as the result shown in Figure 1, we can expect that there should be a relationship between $\alpha_{B O E}$ and $\alpha_{i}$. Indeed, Figure 2 confirms the assumption; the correlation coefficient of the displayed dependence is equal to 0.67 .

As is seen from Figures, the S and M asteroids with moderate albedo have the largest values of $a_{\text {BOE }}, \alpha_{\mathrm{i}}$, and $\alpha_{B O E}$. These values decrease for the E and C types with higher and lower albedos, respectively.

In the future, it is necessary to identify those physical characteristics of the asteroid surfaces that control the BOE amplitude and inversion angle and provide the relationship between them.

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