Binary Asteroid Orbit Sensitivity to Gravity Field Coefficients: Applications to the AIDA Mission Target 65803 Didymos.
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Introduction: As planning for the AIDA mission moves forward, scientists and mission designers will require a more rigorous understanding of the dynamical effects and observability of the gravity field coefficients of the Didymos binary system. This study seeks to provide a baseline correlation between gravity field coefficient expansion order and the dynamical behavior of the system. This is accomplished by implementing the recursive inertia integral formulation of the full two body problem (F2BP) dynamics presented in [1]. While previous F2BP models were limited by the computational inefficiency of their mass models or were not solved for arbitrary expansion orders, the model implemented herein is able to efficiently solve the F2BP up to the Nth order [2,3,4]. We leverage these capabilities to simulate a nominal Didymos system and a series of mass distribution perturbed systems to understand the sensitivity of visual observations of the system dynamics to gravity field coefficients.

Mutual Gravity Potential: The dynamics model implemented herein leverages inertia integrals in the mutual gravity potential to enable Nth order simulation of arbitrarily shaped constant density bodies. A full derivation and explanation of notation is provided in [1], therefore only a summary of the mutual potential is provided. By rederiving the mutual potential with mass distributions in terms of Nth order inertia integrals a recursive summation can be implemented which greatly improves computational efficiency and allows for simulation of arbitrary shape and expansion order. The inertia integrals are the result of a Legendre polynomial expansion used to approximate the mass distribution of each body, as described in Eq.1, and allow the mutual potential to be computed as a series expansion to an arbitrary order of the inertia integrals [5]. Each order N of the inertia integrals consists of a set of 3N inertia integral parameters which represent the full range of axial coupling of the mass distribution analogously to moments of inertia. Computation of the inertia integral parameters from an existing shape model is accomplished in a parallel manner to spherical harmonics, wherein the model is decomposed into a set of tetrahedra with uniform density and the inertia integrals of each tetrahedron are summed to compute the inertia integrals for the given shape model.

The second step in re-framing the mutual potential is to reformulate the series expansion representation into a recursive formulation using the recusion coefficients t_k^n, a^k, and b^{n-k}. This step greatly improves the computational efficiency of the dynamics model while maintaining the simple ordered structure enabled by the inertia integrals. The resulting mutual gravity potential is expressed in Eq. 2 and 3, and is more fully described in [1].

$$T_B^{ijk} = \frac{1}{M_B r^{i+j+k}} \int x^i y^j z^k \, dm \quad [\mathbf{1}]$$

$$U = -G \sum_{n=0}^{N} \frac{1}{R^{n+1}} \tilde{U}_n \quad [\mathbf{2}]$$

$$\tilde{U}_n = \sum_{k(2)=n}^{n} t_k^n \, \Sigma_{(i_1 i_2 i_3)(i_4 i_5 i_6)(j_1 j_2 j_3)(j_4 j_5 j_6)} a^k_{(i_1 i_2 i_3)(i_4 i_5 i_6)} \times b^{n-k}_{(j_1 j_2 j_3)(j_4 j_5 j_6)} e_x^{i_1+i_4} e_y^{i_2+i_5} e_z^{i_3+i_6} \times M_B r^{i+j+k} T_B^{(i_1+j_1)(i_2+j_2)(i_3+j_3)} \times M_{B'} r^{i+j+k} T'^{(i_4+j_4)(i_5+j_5)(i_6+j_6)}_{B'} \quad [\mathbf{3}]$$

where k(2) implies stepping by 2

Observational Accuracy to Order 10: An initial study of the observational sensitivity to higher order inertia integrals was performed, simulating six orbit periods of the nominal Didymos system using order 0 to 10. For this investigation the nominal Didymos system is considered to be a primary body modeled as the AIDA team Didymos radar model and a secondary body modeled as the binary asteroid secondary 1998 KW4b scaled to the ellipsoidal dimensions of Didymos-b. The initial conditions used for the nominal system simulations are the orbit elements presented in the AIDA team reference document with the asteroids initially at periapis and axially aligned along the long axis of inertia [how to cite?].

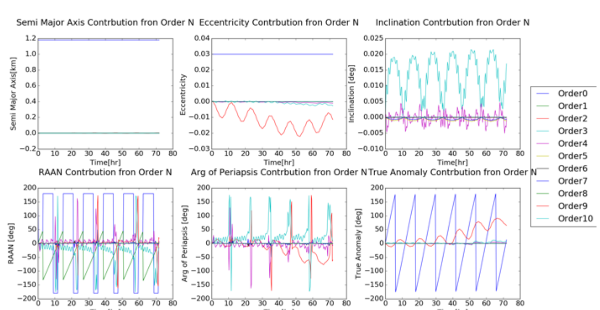

Figure 1: Contribution of order 0 to 10 to the Kepler orbit element behavior of the nominal Didymos system over six orbit periods.

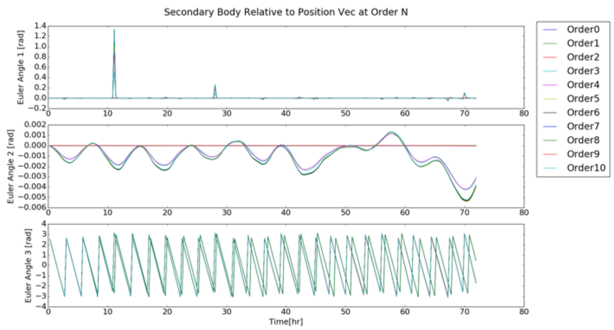

Figure 2: Resulting behavior of order 0 to 10 for the relative attitude between the secondary asteroid principal frame and the separation vector over six orbit periods for the nominal Didymos system.

Simulation results are analyzed based on the convergence pattern of the higher order dynamics towards "truth" behavior. Thus a relationship between simulation order and observable accuracy is developed, wherein the dynamical accuracy gained by each inertia integral order can be related to a per-orbit shift in dynamical behavior of the system. Fig. 1 illustrates observed behavior for the Kepler orbit elements, where each curve represents the contribution of a given order to the behavior of the particular orbit element. The behavior of the orbit elements shows a convergence of the translational dynamics to the centimeter level near order 4 of the inertia integrals. Further analysis of the attitude of the secondary asteroid relative to the separation vector, Fig.2, appears to show similar convergence behavior for the attitude dynamics of the system. The decreasing variation of the dynamics with increasing order illustrated in both figures provides a baseline relation between simulation order and required observation accuracy.

Pertubation of Zero and Second Order Mass Distribution: While higher fidelity simulations provide insight into observation accuracy requirements, it is also of interest to understand how low-order perturbations to mass distribution affect the system behavior. Such simulations both provide further insight into observation requirements and allow for asscociation of specific dynamical behaviors with specific inertia integrals. For our preliminary investigation this is accomplished by perturbing the zero and principle axis second order inertia integrals by 10% of their value in the nominal system described previously. The first order is ignored due to its negligible magnitude and perturbation of higher order terms are not included for the sake of clarity. All simulations for this portion of the investigation were modelled at the fourth order based on the results of previous section. The same initial conditions as the previous section were used for this set of simulations.

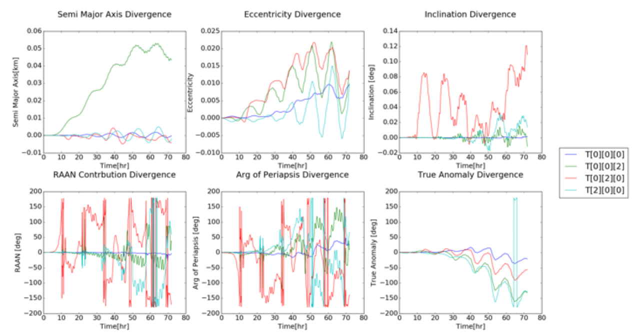

Figure 3: Divergence error of Keplerian orbit elements from nominal system fourth order simulation results

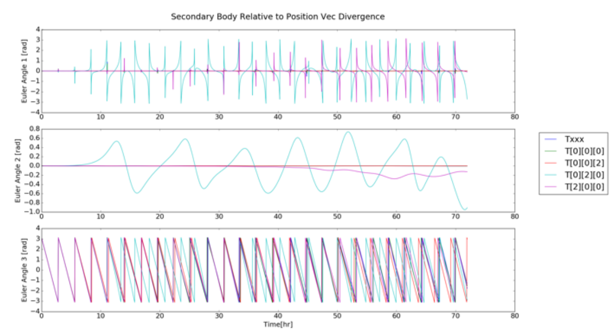

Figure 4: Divergence from the nominal Didymos system over six orbit periods of the relative attitude between the secondary asteroid principle frame and the separation vector.

Simulation results for each perturbed inertia integral were processed and will be analyzed to infer relations between the dynamical behavior and perturbed parameters. A number of unique behaviors associated with each perturbed inertia integral can already be observed in Fig. 3 and 4. The semi-major axis of the T_{002} perturbed system displays growth in semimajor axis that is significantly more rapid compared to the other perturbed systems. Inclination also shows uniquely large amplitude perturbations of the T_{020} system. The T_{020} system also diplays uniquely large amplitudes for the second euler angle, rotation about the y-axis. Unique euler-axis two behavior is also observable for the T_{200} system, which shows very slow growth, that may be periodic. All three of the euler angles, as well as the true anomaly show a growing separation between all the perturbed systems, as is to be expected

References: **[1]** Hou, X., Scheeres, D.J. and Xin, X., (2016) CMDA, **[2]** Ashenberg, J. (2007). CMDA, 99(2) **[3]** Fahnestock, E. G., & Scheeres, D. J. (2006) CMDA, 96(3-4) **[4]** Boué, G., & Laskar, J. (2009) Icarus, 201(2) **[5]** Tricarico, P. (2008) CMDA, 100(4) **[6]** AIM Didymos Reference Document, v. 10, Oct. 22, 2015.