EXPLORING ADVANCED ESTIMATORS FROM GUIDANCE, NAVIGATION AND CONTROL IN FIREBALL MODELLING.

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Introduction: Observing bright meteor phenomena from multiple dedicated fireball observatories can lead to the discovery of a meteorite with a calculated orbit. The Desert Fireball Network (DFN) in Australia has 32 autonomous observatories across outback Australia, covering an area of 2.6 million km^2. In order to handle this 60 Tb/month dataset, a digital pipeline is used to reduce the data. Software that has been developed for this pipeline includes event detection, calibration of coordinates, triangulation, mass determination, orbital calculations, wind modeling and the prediction of a fall line for potential surviving masses.

Determining the mass of the meteoroid at the end of the bright flight trajectory is key to finding any potential meteorites. As a meteoroid passes through the atmosphere it loses mass by both ablation and fragmentation. Calculating the changing mass has typically been based on photometric and dynamic modelling [1] which are strongly dependent on an accurate light curve and do not provide a rigorous analysis of the inevitable errors introduced by uncertainties in the observations and the modelling process. No dynamical method alone is currently able to fully characterise a meteoroid during its trajectory. Sansom et al. [2] have previously applied the dynamical equations of meteoroid flight in a method used in guidance, navigation and control (GN&C)- an Extended Kalman Filter (EKF). This was used to estimate the state (position, mass and velocity) of a meteoroid during its bright flight as well as to ascertain a comprehensive understanding of the errors involved. This algorithm however requires the dynamical equations to be linearised and does not specifically include fragmentation events.

More advanced estimators used in the field of GN&C include the Unscented Kalman Filter (UKF), Interacting Multiple Model (IMM) and Sequential Importance Sampling Particle Filter (SIS PF). These approaches are applied here to the trajectory dataset for Bunburra Rockhole [3].

Dynamical equations: The hypersonic aerodynamic equations that are used to model a meteoroid path through the atmosphere are (after [1]):

$$\frac{dv}{dt} = -\frac{1}{2}K\rho_a v^2 m^{(\mu-1)} + g\sin\gamma_e$$

$$\frac{dm}{dt} = -\frac{1}{2}\sigma K\rho_a v^3 m^{\mu}$$

Where ρ_a is the atmospheric density, v and m the velocity and mass of the meteoroid respectively, g the local acceleration of gravity, γ_e the flight angle from vertical, the ablation parameter σ and the shape density coefficient K. These equations are the base models for all the estimators used here. The position of the meteoroid is observed in time and the velocity inherently calculated. The mass is linked to the velocity through these equations. The constants σ and K however are unknown. These equations are also not a perfect representation of the trajectory; however, this is an advantage of using filters as imperfections in the model are accountable in the process noise.

Unscented Kalman Filter: The UKF is an estimator that allows for a more rigorous approach to handling non-linear equations [4]. Rather than estimating the transformation of the mean and covariances at each time step like the EKF, a UKF represents a Gaussian probability distribution by a set of points. These points are then individually propagated according to the dynamic equations and the mean and covariances are recalculated from these.

As the constants σ and K are unknown, in order to use the UKF, as with the EKF, we must precede this estimator with the dynamical optimisation step described by [2]. For the purposes here, we will use the same entry values as given by these authors for the Bunburra Rockhole meteor. The fit of velocities predicted by the UKF to those calculated from observations may be seen in Figure 1. A comparison between the final covariances estimated using a UKF to those calculated by [2] using an EKF can be seen in Table 1.

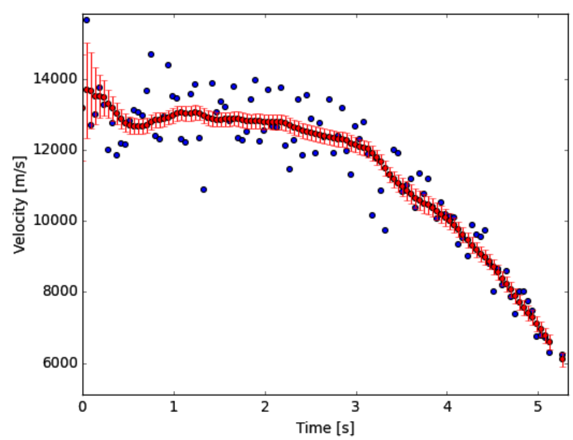

Fig. 1. Velocities calculated from observations (blue) show variable scatter. Velocities estimated by the UKF (red), with covariance as errors, show a good fit.

Interacting Multiple Model: To incorporate instantaneous massive mass loss due to fragmentation, two UKFs may be run simultaneously [5] with different process noise matrices:

1) model parameters with typical error ranges for ablation,
2) model parameters with large errors to include massive fragmentation events.

By using an Interacting Multiple Model (IMM) to weight the likelihood of each model according to the observations, fragmentation events may be handled. This significantly increases the ability to estimate states of the meteoroid proximal to the observations. Figure 2 shows the model 1 is most likely and mass loss is dominated by ablation apart from a brief period at 4.4 seconds where there is most likely a fragmentation event. This can be seen as a spike in the light curve at 34 km altitude on Figure 10 in [3].

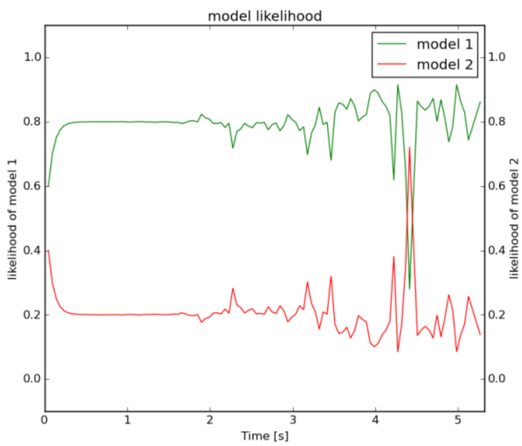

Fig. 2. Model 1 (green) has a low mass covariance and mass loss is dominated by ablation. Model 2 (red) has a high mass covariance and likely indicates fragmentation events when likelihood increases.

Table 1. Comparison of covariance values determined by different estimators for final states.

	Position covariance	Mass covariance	Velocity covariance
EKF	62 m	1.63 kg	241 m/s
UKF	58 m	1.04 kg	203 m/s
UKF IMM	8 m	0.33 kg	61 m/s

SIS Particle Filter: Particle filters apply a Monte-Carlo approach to the filtering problem [6]. This method not only is able to estimate the changing position, mass and velocity of the incoming meteoroid, but also the unknown constants of its flight (σ and K). It is also inherently able to handle fragmentation events, although not explicitly identify their location during the trajectory.

A set of 10,000 particles are initiated with a range of starting parameters that satisfy typical meteoroid characteristics for density, shape, ablation etc. and starting masses ranging from 100g to 5,000 kg. After the prediction of state at t_{k+1} is made for each particle, the resulting state is compared to the observations at that time and assigned a weighting. A new generation of particles is then sampled from this pool based on the weightings. The resulting values of the state (position, mass, velocity, σ and K) are accompanied by an uncertainty matrix.

Discussion and Conclusions: Nonlinear tracking methods may be applied to fireball trajectory modelling to allow a comprehensive understanding of the errors associated with the dynamic equations of this phenomenon. Both Kalman Filters require a single pre-determined set of starting parameters to estimate the states of a meteoroid during its trajectory. The SIS PF does not require a preliminary step to determine starting parameters and takes a Monte Carlo approach to determine the final states based on a broad input range. This approach also means that fragmentation is also accounted for to a certain extent within the covariance ranges.

A powerful tool comes by combining these methods. The use of either Kalman Filter within an IMM allows fragmentation events to be analysed and their timing identified (Fig. 2). We may then run a SIS PF where noise covariances are set to increase at these times. The final values determined by the SIS PF for σ in particular are interesting when this is done as [1] and [3] state that only the apparent value of σ may be determined when fragmentation is not considered. The apparent value determined by [3] for the Bunburra Rockhole meteoroid was an order of magnitude higher than the intrinsic value also calculated by these authors when incorporating brightness (using the MFM method). The value determined by the SIS PF adapted for fragmentation gives the same as intrinsic values in [3]. This indicates that fragmentation is taken into account in the final state estimates. Without brightness data, it has previously been difficult to incorporate fragmentation and determine intrinsic values of σ. Fragmentation events therefore may be incorporated into models without the use of brightness data with a great deal of success.

References: [1] Ceplecha Z. and ReVelle D. O. (2005) *MAPS*, *40*, 35-54. [2] Sansom E. K. et al. (2015) *MAPS*, *50*, 1423-1435. [3] Spurný P. et al. (2012) *MAPS*, *47*, 163-185. [4] Julier S., Uhlmann J. (2004) *Proc. IEEE*, *92*, 401-422 [5] Mazor E. et al. (1998) *IEE T Aero Elec Sys*, *34*, 103-123 [6] Arulampalam M.S. et al. (2002) *IEEE T Signal Proces.*, *50*, 174-188.