A GLIMPSE OF LUNAR CORE SHAPE AND DEEP GRAVITY FIELD. J. G. Williams[1], A. S. Konopliv[1], F. G. Lemoine[2], S. Goossens[2,3], S. W. Asmar[1], R. S. Park[1], D.-N. Yuan[1], D. H. Boggs[1], E. Mazarico[2,4], W. S. Kiefer[5], M. A. Wieczorek[6], M. M. Watkins[1], D. E. Smith[4] and M. T. Zuber[4], [1]Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91109, USA (e-mail James.G.Williams@jpl.nasa.gov); [2]NASA Goddard Space Flight Center, Greenbelt, MD 20771, USA; [3]University of Maryland, Baltimore County, Baltimore, MD 2150, USA; [4]Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA; [5]Lunar and Planetary Institute, Houston, TX 77058, USA; [6]Institut de Physique du Globe de Paris, Paris, 75013, France.

Introduction: The Gravity Recovery and Interior Laboratory (GRAIL) mission accurately determined the gravity field of the Moon [1]. Analyses of the GRAIL mission data found a value for the lunar gravity field coefficient S_{21} that exceeded its uncertainty [2,3,4]. A small nonzero S_{21} means that the principal axes of the moment of inertia matrix are not quite aligned with the reference frame. The misalignment is an order-of-magnitude larger than the frame-generating fits to Lunar Laser Ranging (LLR) data indicate [5,6]. A non-spherical shape for the inner and outer core boundaries can explain the discrepancy. In conjunction with lunar models, the S_{21} value is used to calculate the associated core shape and interior gravity field.

A pole-to-equator flattening of both core boundaries is expected. LLR analysis detects core flattening and other geophysical parameters [6,7] by their affect on the time-varying orientation, the physical librations.

Like the S_{21} coefficient, finite values of C_{21} and S_{22} would imply that the principal axes were misaligned. GRAIL-determined values for both of these coefficientts are smaller and less certain than S_{21}.

Principal Axes: The principal axes of the external gravity field coincide with those of the whole Moon. The Moon has a fluid outer core and the boundaries of that cavity have shape. The principal axes of the outer core may not be aligned with the principal axes of the solid Moon (crust + mantle + inner core). The torques on the solid Moon depend on the whole-Moon gravity field, but the dynamical response orienting the solid Moon depends on the moment of inertia matrix of the solid Moon with its associated principal axes. This distinction has not been considered in past analyses.

The GRAIL analyses, by Konopliv et al. [2] at JPL and Lemoine et al. [3,4] at GSFC, used numerically integrated physical librations, a file of 3-axis orientation vs. time, with initial conditions and many geophysical parameters derived from analysis of LLR data. The three analyses used the numerically integrated physical librations from DE421 [5]. The LLR determination of the principal axis directions for DE421 are thought to be uncertain by about 0.2″ [5,6]. The more recent DE430 file [6] was used for analyses by Konopliv et al. [8]. It benefits from a GRAIL-derived gravity field and its axes are uncertain by about 0.1″. However, both integrations set C_{21}, S_{21}, and S_{22} equal to zero, implicitly assuming that the principal axes of the outer core and the whole-Moon were aligned.

Geometrical and Dynamical Rotations: The unnormalized S_{21} values determined by Konopliv et al. [2,8] and Lemoine et al. [3,4] were about $1.3\text{x}10^{-9}$. The C_{21} and S_{22} values were an order-of-magnitude smaller than S_{21}. By contrast, degree-2 combinations $J_2 - 2C_{22} = 1.58\text{x}10^{-4}$ and $J_2 + 2C_{22} = 2.48\text{x}10^{-4}$ are 5 orders-of-magnitude larger than S_{21} and 6 orders-of-magnitude larger than the C_{21} and S_{22} values.

What rotation of the DE-file axes would align them with the principal axes of the GRAIL-derived, second-degree gravity field? The X axis points near the mean Earth direction, the Z axis points near the mean rotation (polar) axis, and Y completes the triad. For very small right-handed rotations of the frame directions about the X, Y, and Z axes by angles θ_X, θ_Y, and θ_Z,

$$\begin{aligned} \theta_X &= \ \ S_{21}/(\,J_2 - 2C_{22}\,) \\ \theta_Y &= -C_{21}/(\,J_2 + 2C_{22}\,) \\ \theta_Z &= \ \ S_{22}/(\,2C_{22}\,)\,. \end{aligned} \tag{1}$$

The value $S_{21} = 1.3\text{x}10^{-9}$ corresponds to $\theta_X = 1.7''$.

Lunar orientation responds dynamically to torques including those from S_{21}, C_{21}, and S_{22}. Does the dynamical behavior match the geometrical rotations? The first-order analytical technique for calculating perturbations from the gravity field follows Eckhardt [9]. We give the results as small constant rotations about the X, Y, and Z axes by angles δ_X, δ_Y, and δ_Z.

$$\begin{aligned} \delta_X &\approx\ 0 \\ \delta_Y &\approx\ (3/4)\ C^S{}_{21}/(\,J_2 + 2C_{22}\,) \\ \delta_Z &\approx\ -S^S{}_{22}/(\,2C_{22}\,) \end{aligned} \tag{2}$$

Coefficients $C^S{}_{21}$ and $S^S{}_{22}$ use the principal axis frame of the solid Moon. There are three different dynamical responses: 1) There is no significant rotation about the X axis to respond to a finite S_{21} value. The GRAIL-derived S_{21} value corresponds to the geometric θ_X separation of eq. (1). 2) A finite C_{21} value implies that the principal axes of the solid Moon and the whole Moon are separated, but the LLR-derived axes should have rotated around the Y axis to partly offset that separation. The X and Z principal axes of the solid body would not be aligned with the axes of the DE file. The $C^S{}_{21}$ with respect to the solid-body axes would be larger than the GRAIL-derived C_{21}. 3) For rotations about the Z axis, the X and Y axes of the LLR frame should be aligned in the longitude direction with the principal axes of the gravity field and whole Moon, rather than the principal axes of the solid Moon. Within the uncertainties, a zero spacecraft value for S_{22} is expected.

The dynamical prediction for S_{21} was tested by introducing an S_{21} value of 1.3×10^{-9} into a numerical integration of the lunar orbit and physical librations and then refitting the LLR data. The weighted root-mean-square (rms) residual improved 0.1%. Any constant rotation of the lunar frame was small compared to the geometrical rotation of 1.7″. The small S_{21} coefficient has a minor effect on the rotation dynamics.

Core Oblateness: Torque from an oblate core-mantle boundary (CMB) shape is proportional to the difference between the fluid outer core's polar and mean equatorial moments, $C_f-(A_f+B_f)/2$, provided that the fluid has uniform density and the inner boundary is spherical. This moment difference depends on the product $f\,C_f$ of the fluid core moment of inertia C_f and the CMB flattening f, where $f = [C_f-(A_f+B_f)/2]/C_f$ and $f\,C_f/C = [C_f-(A_f+B_f)/2]/C$. The LLR-determined DE430 flattening $f = (2.46\pm1.4)\times10^{-4}$ [6,7] is based on an LLR solution that used a fluid outer core $C_f/C = 7\times10^{-4}$, near the maximum value expected. Smaller cores should give larger values for CMB flattening.

Meyer and Wisdom [10] matched the LLR flattening of the CMB with a theoretical calculation. They assumed that the exterior non-spherical gravity field was formed near the surface and they extended that field downward to the CMB. The shape of the CMB was presumed to deform so that the potential on its surface was constant. We have repeated that calculation and obtained a similar result. However, if one extrapolates the LLR result to a smaller core by holding $f\,C_f$ fixed, then a smaller core would cause a larger flattening and agreement with model calculation degrades. Disagreement implies that the interior field cannot be calculated accurately from the exterior field alone. There may be lateral density variations in the upper mantle. Zhao et al. [11] may have found heterogeneity, a lower S-wave velocity under the PKT. Also, other geophysical processes may influence core shape.

Deep Gravity: If core shape depends on local deep gravity, then the GRAIL exterior S_{21} value may arise from the interior S_{21} field from the core, and that depends on the mantle's interior field. The connection depends on core size. For calculation we adopt model GPM3 [7], which is similar to the Weber et al. [12] model. The resulting interior S_{21} field is shown in Fig. 1. For a given model, all of the deep degree-2 field components have the same radial dependence at depths below the lateral density variations, but the coefficients have different values. The flattening and interior gravity field that are associated with J_2 are larger than the S_{21} core shape and interior field, but precise values for both components are model dependent.

Summary: It is plausible that the lunar S_{21} value determined by GRAIL data analysis comes from the shape of the fluid outer core. The orientation of the solid Moon around its X axis, which is followed by lunar laser ranging, has low sensitivity to an outer core S_{21} shape. By contrast, the principal moments of the solid Moon should be well oriented about the X axis.

LLR continues to detect outer core flattening. Combining GRAIL and LLR results gives a glimpse of core shape and, possibly, deep gravity.

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References: [1] Zuber M. T. et al. (2013) *Science 339*, 668–671, doi:10.1126/Science.1231507. [2] Konopliv A. S. et al (2013) *J. Geophys. Res. Planets 118,* 1415–1434, doi:10.1002/jgre.20097. [3] Lemoine F. G. et al. (2013) *J. Geophys. Res. Planets 118,* 1676–1698, doi:10.1002/jgre.20118. [4] Lemoine F. G. et al. (2013) *EOS Trans. Am. Geophys. Un.,* Fall Meeting, G31B-01. [5] Williams J. G., Boggs D. H., Folkner W. M. (2008), *JPL IOM 335-JW,DB,WF-20080314-001, March 14, 2008*. [6] Williams J. G., Boggs D. H., Folkner W. M. (2013), *JPL IOM 335-JW,DB,WF-20130722-016, July 22, 2013*. [7] Williams J. G. et al. (2014) *J. Geophys. Res. Planets,* submitted. [8] Konopliv A. S. et al. (2014) *Geophys. Res. Lett*., submitted. [9] Eckhardt D. H. (1981) *Moon Planets 25*, 3–49, doi:10.1007/BF00911807. [10] Meyer J. and Wisdom J. (2011) *Icarus 211*, 921–924, doi:10.1016/j.icarus.2010.09.016. [11] Zhao D. et al. (2012) *Global Pl. Change*, *90-91*, 29–36, doi:10.1016/j.gloplacha.2012.01.004. [12] Weber R. C. et al. (2011) *Science 331*, 309–312, doi:10.1126/science.1199375.

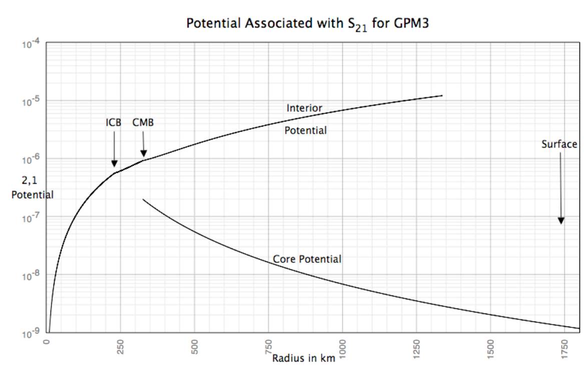

Figure 1. Radial dependence of the interior potential with the S_{21} shape for the GPM3 model structure. Lower curve arises from core shape. Upper curve from crust + mantle + core is unknown for upper mantle.