ON THE FITTING OF NON-LINEAR, EMPIRICAL FUNCTIONS FOR THE FITTING OF MODEL CRATER AGES.

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Introduction and Background: Crater population data are often used for the purpose of estimating the age of a planetary surface. The process of estimating a surface age from craters rests on the concept that the more craters a surface has, the older it is. The relative ages can be converted to model absolute ages by a crater chronology function [*e.g.*, 1, 2]; this relates the flux per year to a time in solar system history for a set crater diameter. In practice, the crater chronology function is defined only for what is termed "$N(1)$," the density of craters with diameters (D) ≥1 km. Often, the crater population that a researcher will use to model a surface age does not include $D = 1$ km craters. The researcher must then use a model crater PF to extrapolate $N(1)$, usually by fitting a PF to a limited diameter range. PF represents an idealized population of craters that formed without factoring in post-formation modification processes and crater erasure. It is typically displayed as a size-frequency distribution (SFD) that shows diameter versus the number of craters (*e.g.*, Fig. **1**). After a model PF is matched to the researcher's measured crater population for a certain diameter range, the PF can be used to extrapolate the density of $D \geq 1$ km craters.

The PF is scaled via a published chronology function [*e.g.*, 1, 2] that describes the crater density of all craters $D \geq 1$ km for a given age ($N(1)$); chronologies are only defined for this (somewhat arbitrary) $D = 1$ km point. An alternative method to estimate ages is to use the number-density of all craters larger than or equal to a certain diameter D in km ("$N(D)$") instead of fitting a range of diameters, and using the PF to scale to $N(1)$ (*e.g.*, measure the density of craters on a surface $D \geq 10$ km, determine the ratio of $N(10)$ to $N(1)$ for the PF being used, scale to $N(1)$, then determine the age of that $N(1)$ from the chronology function).

Both these techniques for age estimation rely on and *require* a PF to relate crater densities to the $D = 1$ km point except in the infrequent case when (a) a researcher is able to measure crater densities that bracket $D = 1$ km (*e.g.*, measuring craters $0.5 \leq D \leq 5$ km) *and* (b) craters at $D = 1$ km have not been modified from their formation population. Ergo, fitting the available crater population data to a selected PF and understanding the uncertainties associated with that are key to assigning model crater ages.

Empirical Functions: There are two main PFs used in the literature – the Hartmann PF ("HPF" [3]), and the Neukum PF ("NPF" [1]). They are both different in shape and philosophy, and a comparison is described in [1].

Power laws were used to describe the size-distribution of the product of catastrophic collision physics [4], and they were adopted by W. Hartmann for the HPF, which has used this form since the mid-1960s [*e.g.*, 3, 5-6]. These were fit from the average of several lunar maria that formed within a narrow time period (≈3.2–3.5 Ga).

In the 1970s and '80s, a second system was developed by a group led by G. Neukum to create the NPF. The NPF is also based on lunar mare counts, but it removes the *a priori* assumption of a power law distribution for a "scientifically neutral" polynomial fit (the latest iteration is an 11^{th}-order polynomial [1]). In addition to maria, Neukum used other terrains to extend the diameter range (Neukum *et al.*, 2001).

Both of these empirical functions have a set shape that remains constant with time and does not shift on the abscissa – the *x*-axis which is crater diameters. Similarly, for any given diameter, each function has a set ratio of which it is greater than or less than another diameter that does not change – *i.e.*, the shape is fixed. Therefore, the important issue when using these is what the amplitude is, or by what value are the densities of craters scaled to get the function to best match the observed crater population data.

Current Methods of Fitting: In general, there is no standard method by which researchers will adjust the model production functions to best match the data – it is not rare to refer to it as "an art," for one must also decide, when fitting, what data range best matches the production function over which to perform the fit.

As such, one technique is simply to adjust the function by eye, scaling it until the production function appears to match the crater SFD over the desired diameter range. Other techniques typically rely on residual minimization, calculating the difference between each SFD datum and the production function value for a particular scaling factor at that diameter. The scaling factor is adjusted until the differences between the data and the production function are minimized. The popular CraterStats and CraterStats2 freeware [7] use IDL software's built-in "CURVEFIT" routine, which uses a gradient-expansion algorithm to compute a non-linear least-squares fit to the PF.

Potential Solutions: In mathematics and statistics, there are numerous methods that have been developed to fit observed data to model functions. We will review the most applicable of these techniques, including those that many in the crater community have not considered – a class of techniques known as "bootstrapping." These are a class of statistics that rely on random sampling with replacement, and they allow assignment of various measures of accuracy and do not require that the model function be "nicely" behaved.

References: [1] Neukum, G. *et al*. (2001). *Space Sci. Rev.* [2] Robbins, S.J. (2014). *EPSL*, doi: 10.1016/j.epsl.2014.06.038. [3] Hartmann, W.K. (2005). *Icarus*, doi: 10.1016/j.icarus.2004.11.023. [4] Hawkins, G.S. (1960). *Astron. J.* [5] Hartmann, W.K., *et al.* (1981). *Basaltic Volcanism Study Project*. [6] Hartmann, W.K. (1999). *Metorit. Planet. Sci.*, doi: 10.1111/j.1945-5100.1999.tb01743.x. [7] Michael, G.G., and G. Neukum (2010). *EPSL*, doi: 10.1016/j.epsl.2009.12.041.

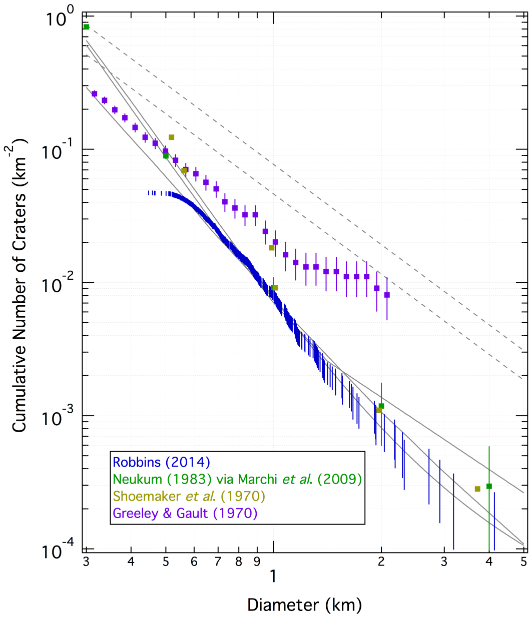

Figure 1: Example population of impact craters identified on and around the *Apollo 11* lunar landing site by various authors (different symbols). Also shown is 3% and 5% of geometric crater saturation, and shown in grey lines are three different model crater production functions. Once the observed population data are fit to a model function by adjusting the vertical offset, a model age could be assigned.