THEORETICAL DETERMINATION OF THE IMPACT CRATER-SIZE-FREQUENCY DISTRIBUTION WITH APPLICATIONS TO MARS AND EARTH. W. Bruckman[1,] A. Ruiz[1],and E. Ramos[2] ; [1]University of Puerto Rico At Humacao,Department of Physics,Call Box 860,Humacao,Puerto Rico, 00792, (miguelwillia.bruckman@upr.edu), [2]University of Puerto Rico At Humacao,Department of Mathematics,Call Box 860,Humacao,Puerto Rico,00792.

Introduction: : A theoretical understanding of the impact crater-size frequency distribution is developed and applied to observed data from Mars and Earth. The analytical model derived gives the crater population as a function of crater diameter, D, and age, τ, taking into consideration the reduction in crater number as a function of time, caused by the elimination of craters due to effects such as erosion, obliteration by other impacts, and tectonic changes. The model is applied to Mars, using Barlow's impact crater catalog[1] (Figures (1)), and we are able to determine a curve ,shown in Figure (2), and Eqs.(1) to (4), describing the number of craters per bin size, $N(D)$, which perfectly reproduces and explains the presence of two well-defined slopes in the $\log[N]\ vs \log[D]$ plot for $D \geq 8km$. For $D \leq 8km$, we see that the theoretical curve differs significantly from the observed data, however, according to Barlow[1], her empirical data undercounts the actual crater population for D less than $8km$ and, therefore, we will restrict our analysis to $D \geq 8km$.

$$N = \bar{\Phi}\tau_{\text{mean}}\{1 - Exp[-\tau_f/\tau_{mean}]\}, \quad 1$$

$$\bar{\Phi}\tau_{\text{mean}} = \frac{1.43x10^5}{D^{1.8}}, \quad 2$$

$$\tau_f/\tau_{mean} = \frac{2.48x10^4}{D^{2.5}}, \quad 3$$

$$\bar{\Phi} = \left(\frac{1}{\tau_f}\right)\int_0^{\tau_f}\Phi d\tau = \left(\frac{1}{\tau_f}\right)\frac{3.55x10^9}{D^{4.3}}. \quad 4$$

Note that $\bar{\Phi}(D)$ is the time average(over the total time of crater formation τ_f) of the rate of meteorite impacts per bin, $\Phi(D)$, capable of forming craters of diameter D. Also, τ_{mean} is the mean-life of craters of diameter D, since it can be shown [2] that $Exp[-\tau/\tau_{mean}]$ is the fraction of craters surviving today, that were formed at time τ ago. We can interpret the above formalism in a statistical or probabilistic manner. Thus, for instance, $\bar{\Phi}$ could be view as a probability of impacts per unit time, while $1/\tau_{mean}$ represents the probability,per unit time, for a crater to disappear. Accordingly, Eq.(1) is the familiar formula describing the evolution in time of $N(D)$, resulting from these production vs destruction processes. It should be emphasized that in general $N(D)$ is not describable by a simple polynomial, like $const./D^{const.}$, but it can be approximate by a combination of them, as we show next.

We see from Eq. (3) that craters with $D \approx 57km$ have $\tau_{mean} \approx \tau_f$, whereas $\tau_{mean} \gg \tau_f,\ if\ D \gg 57km$, and $\tau_{mean} \ll \tau_f,\ if\ D \ll 57km$. Thus, In the limit $D \gg 57km$, $\tau_f/\tau_{mean} \ll 1$, we obtain, from Eqs. (1) and (4), that:

$$N = \bar{\Phi}\tau_f = \frac{3.55x10^9}{D^{4.3}};\ \tau_f/\tau_{mean} \ll 1,\ D \gg 57km, \quad 5$$

which corresponds to a straight line of slope -4.3 in the $\log(N)$ vs. $\log(D)$ plot, that we see in the right-hand part of Figure (2), and is the form of Eq. (1) when we can ignore the destruction of craters. In other words, for these larger craters, their number is simply given by the expected relationship: $N = \bar{\Phi}\tau_f \equiv \int_0^{\tau_f}\Phi d\tau$, when craters are conserved and ,therefore, when the actual crater number is proportional to the age of the underlying surface τ_f. On the other hand, for smaller craters where $\tau_f/\tau_{mean} \gg 1$ we will have, from Eqs. (1) and (2), that

$$N = \bar{\Phi}\tau_{mean} = \frac{1.43x10^5}{D^{1.8}},\ \tau_f/\tau_{mean} \gg 1, D \ll 57km, \quad 6$$

and hence in this limit, N is proportional to the survival mean-life, τ_{mean}, of craters of size D. This feature was called the 'crater retention age'by W. K. Hartmann, and on Mars is shown in craters with D less than about $57km$, corresponding to the straight line segment on the left-hand side of Figure (2) with slope -1.8. Therefore, the above model tells us that the empirical curve is essentially constructed by the two straight lines in the $\log N(D)\ vs \log D$ plot given by Eqs.(5) and (6).The exponent 4.3 is pristine, while the exponent 1.8 is the result of a steady state equilibrium between elimination and creation of craters. The large exponent, 4.3, has interesting implications for the corresponding impactor size-frequency distribution, and we elaborate on this topic below.

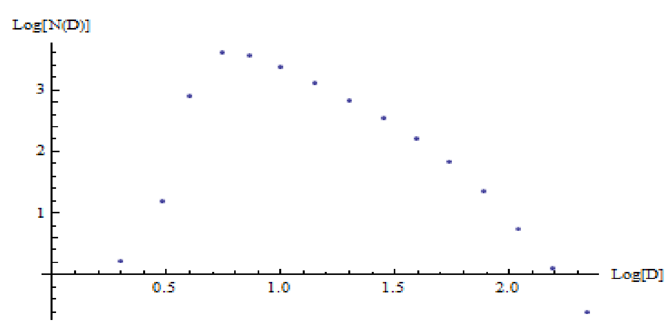

FIGURE (1): Log-Log plot of number of craters per bin, $N(D)\ vs\ D$(km), based on Barlow's Mars catalog. The number $N(D)$ is calculated by counting the number of craters in a bin $\Delta D = D_R - D_L$, and then dividing this number by the bin size. The point is placed at the mathematical average of D in the bin: $(D_R + D_L)/2$. The bin size is $\Delta D = (\sqrt{2} - 1)D_L$, so that $\frac{D_R}{D_L} = \sqrt{2}$.

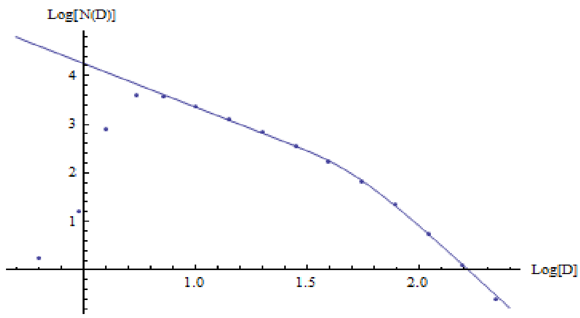

FIGURE (2): Comparing the model in Eqs. (1) to (4) with the Mars data in Figure (1).

We see from Eq. (4) that a numerical calculation of $\bar{\Phi}$ for Mars requires an estimate of τ_f, so with that goal let us write τ_f=$(3.55x10^3/\beta)my,\ my \equiv 10^6$ years, where we expect that β is a number close to 1. For example, the range of values 3000my < τ_f < 4000my is covered by ~0.9 < $\beta < \sim$1.2. Hence, from Eq. (4), we obtain: $\bar{\Phi} = \beta 10^6 (D^{4.3} my)^{-1}$,and thus also find the cumulative rate: $\bar{\Phi}_C$(Mars) $= \int_D^\infty \bar{\Phi} dD = \beta x 10^6 (3.3 D^{3.3} my)^{-1}$. For instance, for $D = 20km$ we obtain: Φ_C(Mars,$20km$)≅ $15\beta (my)^{-1} \approx 15\ (my)^{-1}$, which implies that the cumulative flux per unit area is,:15 $/(4\pi {R_m}^2 my) \cong 100x10^{-9}(mykm^2)^{-1}$, with R_m being the Martian radius, The above results is considerably higher than the values for Earth given by Grieve and Shoemaker[3], namely: $(5.5 \mp 2.7)x10^{-9}(mykm^2)^{-1}$. On the other hand, for $D = 1km$, or, equivalently, impactor energies around a megaton, we have, $\bar{\Phi}_C$(Mars, $1km$) ≅ $(1/3.3 years)$. Therefore, the planification of future Mars exploration, for extended time, should be concerned with meteorites collisions, their associated atmospheric effects, and the expected high speed ejecta.

Let us now study the implications for our planet of a flux of the form: $\Phi = A(D^{-4.3})$,corresponding to the cumulative flux: $\Phi_C(D) = \int_D^\infty \Phi dD = A/(3.3 D^{3.3})$. The value of A can be estimated for Earth from the result of Grieve and Shoemaker[3] for $D = 20km$: $\Phi_C(20km)$= $(5.5 \mp 2.7)10^{-9}(mykm^2)^{-1}\ 4\pi R^2 \approx (2.8/my)[1 \mp 0.50\ ]$, where R is the Earth's radius, and thus obtain: $A = 9.24(20)^{3.3}/my$, which implies

$$\Phi_C(D) = (2.8/my)[1 \mp 0.50]\ ({}^{20}/_{D})^{3.3}]. \qquad 7$$

We have estimated[2] that, according to the above formula, a megaton type impact occurs on our planet approximately every 15 years, while a 10 megatons,tunguska like, blast is expected per century. For these megatons impacts our predictions are higher that most previous estimates, but are in reasonable agreement with updates by NASA and other observations.

An alternative way for the analysis of crater data is the use of cumulative crater-size frequency distribution $\tilde{N} = \int_D^\infty N(D)\, dD$, where $\tilde{N}$ counts the number of craters larger than D. More generally, we can define a matrix $\tilde{N}(D_i, D_i + \Delta D;\ \tau_i,\ \tau_i\ + \Delta\tau)$ counting all craters with diameter between $D_i\ and\ D_f = D_i + \Delta D$, and age between $\tau_i\ and\ \tau_f = \tau_i + \Delta\tau$. The result, given in reference [2], is ,for $\Phi = {}^{A_r}/_{D^m}$ and $1/\tau_{mean} = {}^{B}/_{D^p}$,

$$\tilde{N} = \frac{A_r}{pB}(B\tau_i)^{-n}\{\Gamma[n, \frac{B\tau_i}{D_f^p}, \frac{B\tau_i}{D_i^p}] - (\frac{\tau_i}{\tau_f})^n \Gamma[n, \frac{B\tau_f}{D_f^p}, \frac{B\tau_f}{D_i^p}]\}, \qquad 8$$

where $\Gamma[n, x, y] \equiv \int_x^y x^{n-1}\, e^{-x} dx$, is the generalized incomplete Gamma function, and $n \equiv {}^{(m-p-1)}/_{p}$.

We will illustrate next how these cumulative curves very well describe impact crater data from Earth, although, drastically differ from simple straight line curves in $Log\tilde{N}\ vs\ LogD$, or $Log\tilde{N}\ vs\ \ Log\ \tau$ plots.

The graph below represents $Log\tilde{N}\ vs\ LogD$ for craters older than 10^6 years, from Canada, The Unites States, Europe, and Australia, using "The Planetary and Space Science Centre (PASSC), Earth Impact Database".

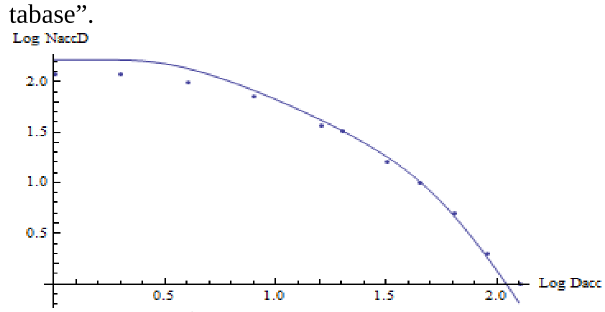

FIGURE (3): $Log\tilde{N}\ vs\ LogD$,for diameters$\geq D$

The theoretical $\tilde{N}(D)$ is in very good agreement with the observations for $D \geq \sim 20km$, although not so good for $D \leq \sim 20km$, which is as expected, due to the undercounting of craters in this interval.

On Earth we can also consider the crater population as a function of age, since reasonable age estimates exist. Thus, in Figure 4 we have the number of craters, $\tilde{N}(\tau)$, older than τ, for the well counted craters with $D \geq 20km$. The very good agreement between theory and observations is noteworthy.

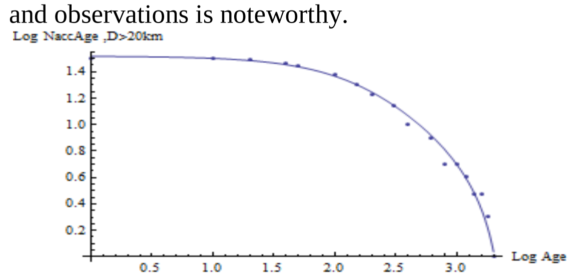

FIGURE (4): $Log\tilde{N}\ vs\ Log\ age\ \tau$

We see that these curves are not well described by straight lines, since their slopes are not constants.

References:[1] Barlow, N.G. (1988) Icarus, 75, 285. [2]Bruckman W., Ruiz A.,Ramos E. .,(2013),arxiv pdf.[3]Grieve and Shoemaker, (1994):The Record of Past Impacts on Earth. In: "Hazards Due To Comets And Asteroids", T. Gehrels, Editor, The University Of Arizona Press.