

**AN EQUATION OF STATE FOR AQUEOUS SODIUM SULFATE USING LOCAL BASIS FUNCTIONS, WITH APPLICATIONS TO OCEAN WORLDS.** R. James<sup>1</sup>, S. D. Vance<sup>2</sup>, and J. M. Brown<sup>3</sup>, <sup>1</sup>Occidental College ([jamesr@oxy.edu](mailto:jamesr@oxy.edu)), <sup>2</sup>Jet Propulsion Laboratory, California Institute of Technology ([svance@jpl.nasa.gov](mailto:svance@jpl.nasa.gov)), <sup>3</sup>Department of Earth and Space Sciences, University of Washington, Seattle ([brown@ess.washington.edu](mailto:brown@ess.washington.edu)).

**Introduction:** The thermodynamics fluids in ocean worlds determine how much ice is present and how the oceans transport materials and heat [1,2,3]. Sodium sulfate is identified as a possible ocean constituent on the basis of chemical models and observations [4]. Detailed models of the influence of salinity in ocean worlds rely on available thermodynamic data. To date, models of high-pressure chemistry in ocean worlds have considered only  $\text{MgSO}_4$  [5,6]. Here, we develop a new equation of state for  $\text{Na}_2\text{SO}_4$  incorporating available measurements of sound speed, density, and specific heat capacity.

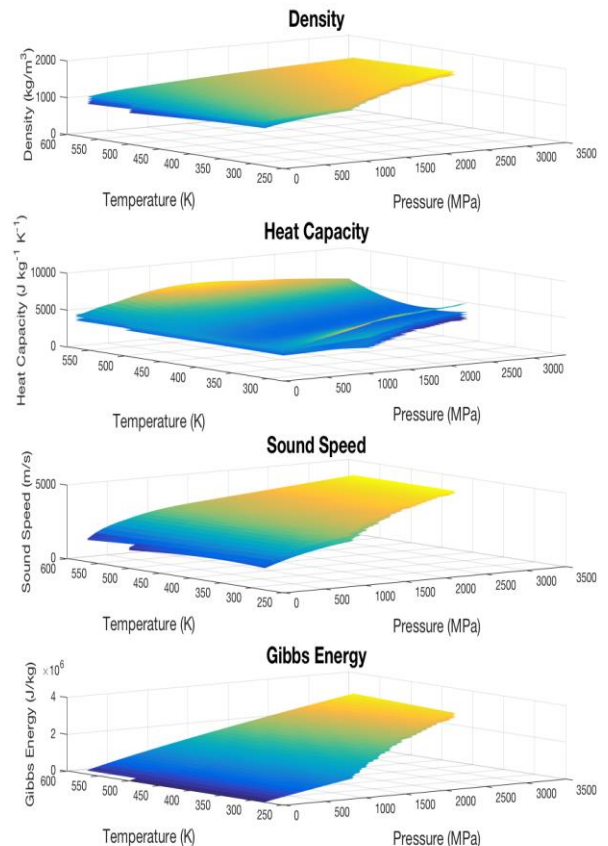
#### Creating the Equation of State:

**Basis Splines.** We use basis splines, or b-splines, to construct a regularized fit to available sound speed measurements. Conventional equations of state use weighted polynomials to create semi-localized fits, which fail to decouple the influence of one region in P, T, and m from the behavior in other regions. B-splines solve this problem, allowing new data to be added trivially without needing to refit the original data.

**Creating the Spline.** Using the b-spline approach, sound speed data from Wiryana [7] and Mantegazzi [8] were fit to surfaces in P, T, and m. The global fit in P and T revealed systematic error in pressure measurements by [8], which was corrected with a constant offset in pressure along each isotherm.

**Equation of State.** Using the sound speed surfaces created from the data, surfaces for density, heat capacity, and Gibbs energy were created using a recursive equation of state algorithm. These surfaces were then used the Matlab implementation [9], `spgft`, to fit the surfaces to intermediate concentrations, and to create the comprehensive equation of state for the  $\text{Na}_2\text{SO}_4$  (aq).

**Equation of State Accuracy and Applications:** Output densities as partial molal volumes match well with the available densities and partial molal volumes for  $\text{Na}_2\text{SO}_4$ . The equation of state encompasses a pressure range beyond the 1.6 GPa relevant to Ganymede, temperatures from the freezing point to 600 K and concentrations up to 1.5 mol  $\text{kg}^{-1}$ . Work in progress is formulating equations of state for crystalline salts and for water ices to more accurately and comprehensively compute phase equilibria and haline mixing in ocean worlds.



**Figure 1:** Density, heat capacity, sound speed, and Gibbs energy for aqueous  $\text{Na}_2\text{SO}_4$ .

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